A11101 294979

NAT'L INST OF STANDARDS & TECH R.I.C.

A11101294979

/Bulletin of the Bureau of Standards
QC1 .U5 V10;1914 C.2 NBS-PUB-C 1905

CONSTANTS OF SPECTRAL RADIATION OF A UNIFORMLY HEATED INCLOSURE, OR SO-CALLED BLACK BODY, I

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I. INTRODUCTION

In the present research two distinct methods are employed in the investigation of the partition of energy in the spectrum of a uniformly heated inclosure or so-called black body. These methods employ (1) "isothermal" and (2) "isochromatic" energy measurements.

Owing to unavoidable delays in assembling all the apparatus for making the measurements at the highest temperatures, using a vacuum furnace, the intention of withholding[1] all the data until the completion of this investigation has now been abandoned. The present paper deals primarily with the form of the isothermal spectral energy curve, extending to about 6.5μ, and with the mathematical equation representing this energy distribution. An exact knowledge of the form of the spectral energy curve is necessary in order to determine whether the computations of the constants should be made on the basis of the Wien or the Planck equation. To obtain this isothermal spectral energy curve the temperature of the radiator must be kept constant while exploring the complete spectrum. In the isochromatic method, energy measurements are made in a selected region of the spectrum, using two or more different temperatures.

In a subsequent paper it is purposed to give what is hoped to be more precise values of the constants of spectral radiation, determined by the method of isochromatic energy curves (also isothermals, obtained with quartz and with fluorite prisms), which method apparently admits of a higher precision when the form of the energy curve, and hence the basis of computation have been established.

It has, of course, been realized all along that the method of obtaining the constants from the isothermal energy curves is subject to numerous difficulties, owing to atmospheric absorption in the apparatus. But it has favorable features not possessed by the method of isochromatics, and it is hoped that it will be possible to place the entire optical path and radiator in vacuo, in order to obtain the spectral energy curve free from atmospheric absorption bands. This will be the crucial test to determine the form of the energy curve. One of the causes of disagreement in the results obtained by present-day experimenters is owing to the method of computation (Planck or Wien equations) employed in the reduction of the observations. With the optical path in a vacuum there is no objection to having a focal length of several meters for the objective mirror, thus increasing the dispersion and hence removing one of the present objections to the use of a fluorite prism. This is especially desirable when working at high temperatures. Using a larger dispersion it will be possible to determine the constants with greater precision by both the isothermal and the isochromatic curves. This no doubt will require a redetermination of the refractive indices of fluorite.

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[1] As stated in J. Wash. Acad. Sci., 3, p. 10; 1913. This is an abstract of the data and manuscript compiled to Dec. 12, 1912. The mean value of C as given here for the "1911" series is high, owing to the large percentage of curves that were observed at high temperatures where the correction for reducing the temperatures above 1400°, observed with a thermocouple, to the gas thermometer scale was about 4° larger than is employed in the present paper.

It may seem somewhat belated to question the prevailing notion that the form of the spectral energy curve has been thoroughly established. However, when one inquires thoroughly into the matter it is found that there is hardly a single piece of experimental work that stands the present-day criteria for deciding whether either one of the formulæ, Wien's or Planck's, fits the experimental observations. In the earlier work of Paschen,[2] the Wien equation seemed to agree with the experimental results. However, viewed from present-day experience this was probably owing to the lack of blackness of the radiators, and the presence of atmospheric absorption which would seriously affect the shape of his energy curves obtained at low temperatures (100° to 500° C).

The results published by Lummer and Pringsheim [3] showed that the theoretical formula of Wien did not fit the experimental curve and a more complicated formula [4] was published which fitted the experimental data at hand when using a certain dispersion curve of fluorite. This curve now appears to be incorrect; hence the formula is incorrect. In a subsequent paper they [5] tested the Planck as well as the Wien equation for the spectral region of 12μ to 18μ and concluded that neither of these formulæ fit the experimental observations. Subsequent to this work, Paschen [6] published data purporting to prove that the Planck equation fitted the observed energy curves, but, as will be mentioned elsewhere, this was made decisive by multiplying the observed curves, beyond 3.9μ, by arbitrary factors, which did not appear to seriously affect the computations of the constants. The comments made at that time, by Lummer and Pringsheim,[7] were to the effect that the results were not a satisfactory proof of the Planck equation.

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[2] Paschen: Sitzber. Akad. Wiss., Berlin, **21**, pp. 403, 959; 1899.

[3] Lummer and Pringsheim: Verh. Phys. Gesell., **1**, pp. 33, 215; 1899.

[4] Lummer and Jahnke: Ann. der Phys. (4), **3**, p. 283; 1900.

[5] Lummer and Pringsheim: Verh. Phys. Gesell., **2**, p. 163; 1900.

[6] Paschen: Ann. der Phys. (4), **4**, p. 277; 1901.

About the only precise work done at that period was that of Rubens and Kurlbaum,[8] who examined the residual rays (9 to 50μ) reflected from quartz, fluorite, and rock salt, the radiator being heated to various temperatures from −188° to +1500° C. They found a closer agreement between the experimental observations and the values obtained by computation using Planck's equation than obtained with any of the other formulæ tested, but there was not an exact agreement.

While the Planck formula was therefore fairly well proven experimentally for the extremely long wave lengths, and therefore presumably for the intervening spectral region, there was evidently room for experimental work on the determination of the exact form of the spectral energy in the region transmitted by a fluorite prism, viz, from the visible spectrum to 6μ in the infra-red.

The most definite statement that can now be made is that the Planck equation fits the experimental observations better than any other equation, which is also based upon plausible theoretical assumptions. But we do not know with certainty whether the theoretical curve will be found to coincide exactly with a perfect experimental energy curve. Planck's introduction of the expression $(e^x - 1)^{-1}$ instead of e^{-x}, which occurs in the Wien equation, raises the emissivity on the long wave-length side of the spectral energy curve, thus bringing it closer to the observed curve. However, some of apparently the best experimental curves (judged by the method of computation of λ_m from equation (6) which should give uniform values) seem to fall somewhat below the theoretical, Planck curve, for the region beyond 5.5μ, just as though the true formula were something lying between the Planck and the Wien equations and having a factor, say $(e^x-0.8)^{-1}$, instead of $(e^x-1)^{-1}$. Just how much of this depression of the observed curve, below the theoretical curve, is caused by atmospheric water vapor can not be determined without an elaborate outfit from which the water vapor and air can be entirely removed. Not until then can this question of the form of the mathematical equation be definitely settled.

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[7] Lummer and Pringsheim: Ann. der Phys. (4), **6**, p. 192 (see p. 210); 1901.

[8] Rubens and Kurlbaum: Ann. der Phys. p. (4), **4** p. 649, 1901.

II. SPECTRAL RADIATION FORMULAS AND METHODS OF COMPUTATION OF THE FORMULAS[9]

The great mass of theoretical speculations, now at hand, seem entirely out of proportion to the slender experimental data upon which they are based. It is therefore desirable to proceed to the discussion of the present experimental research without further reference to theory.

In the previous researches already cited the position of the maximum emission, λ_m, of an isothermal spectral energy curve appears to have been computed by taking the wave lengths, λ_1 and λ_2, corresponding to equal emissivities, $E_1=E_2$, on the normal spectral energy curve. The formula used for computing λ_m is:

$$\lambda_m=\frac{(\log \lambda_2-\log \lambda_1)\ \lambda_1\ \lambda_2}{(\lambda_2-\lambda_1)\ \log e} \tag{1}$$

which is obtained directly from the Wien equation.

$$E_\lambda=c_1\ \lambda^{-a}\ e^{-c/\lambda T} \tag{2}$$

The Wien displacement law, which is the mathematical expression for the shifting of the maximum emission toward the shorter wave lengths, with rise in temperature, is:

$$\lambda_m T=A \text{ (a constant).} \tag{3}$$

The important spectral radiation constant, in equation (2) useful in optical pyrometry, etc., is:

$$C=\alpha\lambda_m T \tag{4}$$

in which the value of $\alpha=5$.

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[9] In order to have a convenient and precise nomenclature, the following symbols are used: A for $\lambda_{max}\ T$; λ_m for λ_{max}; C for c_2 of former publications; σ for the coefficient of total radiation.

There are, of course, other methods for determining the radiation constant, C; but for the purpose of the present investigation it seemed better to obtain a great many computations of this constant, from the most reliable parts of an isothermal spectral energy curve (which is possible with equations (1) and (6)) than to attempt to determine C from a determination of the slope of an iso-chromatic curve. Isochromatic energy measurements were, of course, also made with the same apparatus, but fluorite is not well adapted for the purpose. Furthermore, since the computation of that data is incomplete, the discussion of the results will be given in a forthcoming paper, as already mentioned. At present this procedure of obtaining mainly isothermal curves seems the best that could have been followed, considering the difficulties which arise in operating a vacuum bolometer and maintaining it at a constant radiation sensitivity.

From the beginning it was found that, when computing λ_m by equation (1), the values usually decreased uniformly when λ_1 and λ_2 were taken at equal ordinates (emissivities, $E_{\lambda 1} = E_{\lambda 2}$) which were far apart, to a fairly constant value for λ_1 and λ_2 taken close together. This condition had not been mentioned by previous investigators. Hence, when the first announcement was made [10] of the progress in the present investigation (recorded in the present paper as "series of 1909") no values of the constants were given other than the general statement that the constant A, in equation (3) seemed to be as high as any value that had been published at that time. This high value, as is now evident, was owing partly to the method of computation by equation (1). There was also evidence that radiation could come from the cooler parts of the porcelain radiator, which became soft and sagged when operated at high temperatures. This would also tend to give too high values to the constants, and the work was undertaken anew.

It was then decided to compute the value of λ_m from Planck's equation

$$E_\lambda = c_1 \lambda^{-a} (e^{c/\lambda T} - 1)^{-1} \qquad (5)$$

on the assumption that the observed curve fits the theoretical curve, which is now found to be true for about 80 per cent of the experimental curves. The value of λ_m is easily derived[11] from equation (5) by equating $E_{\lambda_1}=E_{\lambda_2}$, and for the complete solution it is:

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[10] Amer. Phys. Soc. Proc., Apr. 26, 1909; Phys. Rev., 28, p. 466, 1909.

$$\lambda_m=\frac{\alpha\,(\log\lambda_2-\log\lambda_1)\lambda_1\lambda_2}{\alpha'(\lambda_2-\lambda_1)\log e}-\frac{\lambda_1\lambda_2\left[\log\left(1-e^{-c/\lambda_1 T}\right)-\log\left(1-e^{-c/\lambda_2 T}\right)\right]}{\alpha'(\lambda_2-\lambda_1)\log e}\quad(6)$$

The second term in this equation can usually be abbreviated, since terms involving λ_1 are usually negligible. For values of λ_2 which are less than 4μ the term $\log\left(1-e^{-c/\lambda_2 T}\right)$ may be expanded into a series[12] and (by dropping all terms but the first) be used in the form $-e^{-c/\lambda_2 T}\log e$. In this equation $\alpha'=4.9651$ and $C=\alpha^1\lambda_m T$. It may be noticed that an approximate value of C is necessary in order to obtain a solution for the second-term factors in equation (6). This may be obtained by solving for λ_m by using only the first term in equation (6). All the final results of the present research have been reduced by using a value of $C=14500$ in computing the value of the second term in equation (6). However, in computing the λ_m from any spectral energy curve, a value of $C=14\,400$ would decrease the mean value of λ_m by only about 0.005μ, for the temperatures at which the present investigations were made.

In case one can not observe the complete energy curve—as, for example, when using a quartz prism which absorbs heavily beyond 2.2μ—it is possible to compute the value of λ_m from the Planck equation by reducing the observed energy curve to the normal spectrum and taking from the energy curve the values of E_1 at λ_1 and E_m. The proper equation is:

$$\frac{E}{E_m}=\left(\frac{\lambda_m}{\lambda}\right)^5\frac{\left(e^{4.9651}-1\right)}{\left(e^{4.9651\lambda_m/\lambda}-1\right)}\quad(7)$$

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[11] Phys. Rev., **32**, p. 591, 1911. Unfortunately, as first published (Phys. Rev., **29**, p. 553, 1909, and Jahrb. Radioaktivität, **7**, p. 123; 1910), this solution contained numerous typographical errors. Although never so used, the second term was then published in a more symmetrical form, which is, however, more cumbersome for some methods of computation. The statement in a subsequent note (Phys. Rev., **31**, p. 317, 1910) that this method of computation is illusory in spite of its applicability is erroneous. After eliminating an error in the numerical calculations, this method of computation is now found to be the only one which gives concordant results.

[12] This is shown in a paper by Buckingham and Dellinger, this Bulletin, **7**, p. 393, 1911. In practice, however, it is found that such approximate solutions are inadequate for wave lengths greater than 3.7 to 4.0μ, and there is but little time saved in so using the formula.

This equation appears to be well adapted when using a large dispersion, as, for example, when using a quartz prism; but the temperatures must be sufficiently high so that the E_m does not fall in the region beyond 2.2μ where quartz begins to absorb heavily. This equation is not well adapted to a fluorite prism, owing to the steepness of the energy curve on the short wave-length side, where one would want to use it, and also owing to the fact that, at the temperatures usually employed in the present research the E_m usually falls in the region of atmospheric absorption bands or in the region of 1.6μ where the dispersion curve of fluorite has a point of inflection. The latter, as explained elsewhere, makes it quite difficult to determine accurately the factors for reducing the observations from the prismatic to the normal spectrum. This equation has therefore not been used in computing λ_m from the curves obtained with the fluorite prism. With a larger dispersion, as already mentioned, equation (7) could be used as a check on equation (6).

For computing the constant, C, from an isochromatic energy curve, at any wave length, λ, Planck's equation was used in the following form:

$$c = \frac{(\log E_2 - \log E_1)\lambda T_1 T_2}{\log e (T_2 - T_1)} - \frac{\left(e^{-c/\lambda T_2} - e^{-c/\lambda T_1}\right)\lambda T_1 T_2}{T_2 - T_1} \qquad (8)$$

where E_1 and E_2 refer to the emissivities corresponding to the temperatures T_1 and T_2, respectively. In this equation the terms $\log \left(1 - e^{-c/\lambda T_1}\right)$ etc., which are similar to the terms in equation (6) are expanded in series and only the first term of the expansion $\left(-e^{-c/\lambda T_1} \log E\right)$ is used. As in equation (6) an approximate value of $C = 14\ 500$ is used in applying the second term correction. For wave-lengths up to 1μ this correction term (to the Wien equation) is small, being only 2.1 and 4.3 for temperature intervals of $(T_2 - T_1)$ 363° and 623°, respectively, when using $T_1 = 1450$. However, these corrections increase very rapidly with wave length beyond 1μ, so that at 2μ, with the same temperature intervals ($T_1 = 1450$; $T_2 = 1813$ and 2073°) the corrections to the values of C amount to 168 and 227, respectively. This explains the rise in the value of C with wave length as found by Lummer and Pringsheim, who did not use the second term correction. Energy curves which coincide closely with the Wien equation would give uniform values of C with increase in wave length, when computed by using only the first term in equation (8), as was found by Paschen. In their preliminary results Warburg[13] and his assistants found that the value of C, obtained from isochromatics using a quartz prism, is independent of the wave length. On the other hand, they found an increase in C with wave length when using fluorite prisms.

A geometrical solution for λ_m proposed by Buckingham [14] was applied to the series of observations [15] recorded in this paper as the "Series of 1910." The computations, based upon this geometrical method for applying the second term correction factor in equation (6), do not differ much from the present computations in which the complete correction factor (second term in equation (6)) was applied. However, the geometrical method for applying the second term correction factor is not entirely free from arbitrary assumptions, so that all the herein-described observations were recomputed on the new basis of equation (6), with additional corrections for losses by reflection at the prism faces, and the fluorite window covering the bolometer. These additional corrections, occurring as they do on the short wave-length side of the energy curve, should and do give somewhat smaller values of λ_m for the 1910 series than was obtained by the approximate solution. The newly computed values of the 1910 series are in agreement with all the others, as now computed on the basis of the Planck equation. From the beginning it was felt of the greatest importance to obtain an extensive series of observations, under all sorts of conditions, leaving the exact computation of the results until the very last and then computing all the data on a uniform basis. From the results obtained this procedure seems amply justified. The fact that the numerical values of the constants are smaller than the older determinations of Paschen, and of Lummer and Pringsheim (and, for that matter, the earlier values of the present data, obtained by a different system of computation, and not including all the correction factors for reflection) need not disconcert us; for as noted elsewhere the older data are in agreement with the present values, when computed on the same basis. Ultimately it may be proven that the spectral energy curves fall between the theoretical ones represented by the Wien and the Planck equations which would give higher values to the constants. Only a determination of the complete spectral energy curves, entirely free from absorption bands, will decide this question. As already mentioned, this will require the placing of the entire optical system in a vacuum. In the meantime it is gratifying to note that from 75 to 80 per cent of the spectral energy curves, obtained in the present investigation, are found to fit the Planck equation within the experimental errors of observation.

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[13] Warburg, Hupka, and Müller: Zs. Instrk., **32**, p. 134; 1912.

[14] Now incorporated in a paper by Buckingham and Dellinger, this Bulletin, **7**, p. 393; 1911.

[15] Phys. Rev., **31**, p. 317; 1910. Jahrb. Radioaktivität und Elektronik, **8**, p. 1; 1911. The mean value of energy curves gave a value of $\lambda_m = 2944$.

Other criteria might have been used to test the validity of the Planck equation; for example, the increase in the maximum emission ($E_m = BT^\alpha$) with rise in temperature. But this involves uncertainties owing to the maximum emission shifting into atmospheric absorption bands, variation in the radiation sensitivity of the apparatus, etc. Furthermore, it puts the burden of the proof upon the temperature and not the radiation measurements. The computation of λ_m by equation (6) is less arbitrary, for it permits the selection of the best parts of any given energy curve, where there are no absorption bands; and the number of computations on any curve is limited only by the patience one has in making them. It certainly speaks well for the method (by equation (6)) that values λ_m computed from values of λ_2, extending from 2.5μ to 5.5μ, are in agreement to 1 part in 200, as was often found when the energy curves were sufficiently free from absorption bands to permit computation in the doubtful region at 2.5μ. No computations were made by bridging over absorption bands, and then reading values of λ_1 or λ_2 falling therein. Owing to the great number of energy curves, most of which have been recomputed and redrawn during the past six months, it was not possible to spend any length of time in studying the curves. This might have produced a series of values which differ less from the mean value. However, the great mass and diversity of the material obviates the personal bias which can so easily, though unconsciously, enter work of this type.

III. SUMMARY OF PREVIOUS INVESTIGATIONS

In spite of all that has been published on the partition of energy in the spectrum of a black body there is but little experimental data at hand which is more than qualitative in value.

Paschen[16] observed an extensive series of spectral energy curves at temperatures ranging from 100° to 450° C, using six different kinds of bolometers covered with different kinds of absorbing material, e. g., lampblack and platinum black or copper oxide; also having the bolometers in the focus of a hemispherical mirror. The radiators were heated by means of boiling water, aniline, sulphur, etc. The mean value of all his observations, which are in close agreement, was $A = 2890$. He continued the work at temperatures ranging from 400° to 1050° C using a large porcelain radiator. He used also metal cups of copper or platinum blackened with oxides of copper or iron, and heated within this porcelain radiator, making in all about a dozen different arrangements of the radiators. The bolometer was covered with platinum black and was situated in the focus of a hemispherical mirror to "blacken" it. The mean of the new series was $A = 2907$, with a probable error of ± 16. The mean value of all his work up to this time is about $A = 2900$, and since the contours of his energy curves were supposed to fit the Wien equation ($\alpha = 5$) the value of C is 14 500. In the meantime Lummer and Pringsheim (their work will be discussed presently) came to the conclusion that the Wien equation did not fit their experimental results. Paschen[17] thereupon undertook the work anew, after redetermining the reflecting power of silver and the refractive indices of fluorite. He used a porcelain tube radiator, also three other radiators which he blackened as in the preceding research. He took the wise precaution to project an image of the radiating wall of the black body upon the spectrometer slit in order to avoid possible radiations from the side walls and diaphragms falling upon the prism and bolometer. He made complete corrections for the selective reflection of the silver mirrors and also for the loss by reflection from the prism. He then found that his observations fitted neither the Wien nor the

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[16] Paschen: Sitzber, Akad. Wiss. Berlin, **21**, pp. 403, 959; 1899.

[17] Paschen: Ann. der Phys. (4), **4**, p. 277; 1901.

Planck equation, the values on the long wave-length side of the energy curves falling between the two theoretical curves. He found that if (for reasons he himself could not explain, loc. cit., p. 295) he multiplied his observations by factors varying from 1.02 at 3.91μ, 1.038 at 4.59μ, 1.050 at 5.12μ, 1.068 at 6.26μ, 1.195 at 8.25μ, etc., the observed energy curve would fit the Planck equation and fulfilled better the condition of congruence. Upon this basis, and presumably by computing the λ_m by using equation (1), he obtained a value of $A = 2921$ for his newest data. He then applied factors to some of his previous data, thus making them agree better with the Planck equation and the value of A was increased from 2890 to 2915, or about 0.87 per cent. From this it appears that, if he had not multiplied his observations by these arbitrary factors, his latest results would have been about 0.87 per cent lower or $A = 2900$, which is practically the same as the value previously obtained.

TABLE 1

Data from Paschen

Temp. absolute	λ max. (Wien eq)	λ_m T	λ^1 max. (Planck eq)	λ^1_m T
	μ		μ	
1580.6	1.846	2918	1.837	2903
1567.9	1.858	2913	1.850	2901
1357.4	2.157	2929	2.150	2918
1352.7	2.159	2920	2.152	2911
1347.4	2.172	2927	2.164	2916
1337.4	2.188	2926	2.180	2916
1055.2	2.777	2929	2.769	2922
1053.7	2.770	2919	2.762	2910
1052.2	2.781	2926	2.774	2919
875.0	3.328	2911	3.319	2904
876.4	3.333	2914	3.324	2913
Mean..........	....	2921	....	2912±5

C=14,458±25

We can obtain a further and more plausible estimate of the correction to Paschen's values by applying the values of the second term which result from computing λ_m by equation (6) in the present research. This is admissible because his observed curves are said to fit the Planck equation to about 4μ. His energy curves can not, therefore, differ much from those obtained in the present research for the spectral region up to 4μ. Proceeding on this basis the values of λ'_m (see column 4) in Table 1 were obtained by multiplying Paschen's values of λ_m (column 2) by 1.007 (i. e., α/α') which transfers them from the Wien to the Planck system of computation. From these new values of λ_m were deducted the second term factors of equation (6), which were obtained from the computations of the spectral energy curves, observed at closely the same temperatures in the present research. The second term correction factors were taken from computations in which λ_2 fell in the region of 3.4 to 3.9μ, where the value of λ_m, as computed from the present research by using the first term in equation (6), has closely the same value as published by Paschen. The minimum value of this second term factor for this spectral region varied from 0.022μ at the highest temperatures to 0.033μ at the lowest temperatures. However, second term corrections which are 5 to 10 per cent higher would not seriously affect (decrease) the results. Column 4 of Table 1 therefore gives a fair estimate of the values of λ'_m, on the basis of computation required by Planck's equation. This gives a mean value of $A = 2912$ and $C = 14\,458$, which is close to the value obtained by the rough estimate of the correction, and it happens to be the same as found in the present experiments.

Since the original values at long wave lengths were arbitrarily changed by Paschen to fit the Planck curve, the writer feels justified in attempting to reduce the data in this manner by the methods involved in the present research. On this basis, and with the clue that the curves fitted the Planck equation to about 4μ, it appears that, if Paschen had used the present system of computation, his value of C would be of the order of $C = 14\,458$.

The most reliable values of the constants published by Lummer and Pringsheim[18] require a treatment similar to that just given to Paschen's data. They used the earlier indices of refraction of fluorite, published by Paschen[19] in 1894, which are in error in wave length by 0.02μ for the region of 1 to 2.5μ. But little information is at hand as to how they obtained their values of λ_m. In their earlier work they say that, after bridging over the absorption bands by means of a smooth line, the values of λ_m and E_m may be read directly from the spectral energy curve.[20] Making the corrections to their values λ_m (column 4, Table 2), which, as already stated, are too large by about 0.02 μ at 1.7 to 2.7 μ, owing to a wrong calibration curve for the fluorite prism, their mean value of A is reduced from 2940 to 2930. Their curves did not fit the Wien equation, and the Planck equation was not yet published in its complete form when they made known their results. From the present research, using similar radiators of their design under similar conditions, it is evident that their energy curves can not be markedly different from these obtained by the writer. Their curves should therefore be reduced by the present system of computation in order to make the constants comparable with the values obtained in the present research. This was accomplished by multiplying their corrected values of λ'_m by 1.007 $(=\alpha/\alpha')$ and by applying the second term correction of equation (6). The values of A obtained by the system of computation which is required by Planck's equation, are given in column 5 of Table 2. The mean value is $A=2911$ and $C=14\ 450$. If we exclude the last value of $A=2814$, which is evidently not comparable with the

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[18] Lummer and Pringsheim: Verh. Phys. Gesell., Berlin, **1**, p. 215, 1899.

[19] Paschen, Ann. der Phys. (3), **53**, p. 301, 1894; (4), **4**, p. 299, 1901.

TABLE 2

Data from Lummer and Pringsheim

Temp. absolute	λ max. (Wien eq)	λ_m T	λ^1max. (Planck eq)	λ^1_m T
	μ		μ	
1646.0	1.78	2928	1.745	2871
1460.4	2.04	2979	2.010	2933
1259.0	2.35	2959	2.301	2896
1094.5	2.71	2966	2.690	2943
998.5	2.96	2956	2.945	2940
908.5	3.28	2980	3.260	2961
723.0	4.08	2950	4.050	2927
621.2	4.53	2814	4.530	2814
Mean		2940		(2911) 2924±8

C=14,518±40

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[20] Lummer and Pringsheim: Verh. Phys. Gesell, 1, p. 33, 1899.

rest, the mean value is $A = 2924$ and $C = 14\,518$. They made no correction for loss by reflection from the silver mirrors and from the prism, which would be most effective in shifting the energy curve on the short wave-length side of the maximum toward the visible spectrum, and would still further decrease their value of C by 0.3 to 0.4 per cent. Hence, a fair estimate of the constant as obtained by Lummer and Pringsheim is $C = 14\,460$ to $14\,470$. Since their porcelain tube radiator was supported only at the ends and hence would bend in the middle when heated above 1200°, and since this radiator was placed directly before the spectrometer slit (judging from the writer's experience with a similar outfit), it was impossible to be certain that no radiation came from other parts of the black body than the central radiating wall of the tube. Radiation from the cooler diaphragms tend to give too large a value of λ_m for the temperature indicated by the radiating wall.

Unfortunately none of their original observations were published. Values for a few points of two spectral energy curves were published[21] in comparing the experimental results with theoretical values computed by various radiation formulæ. Constructing the curves for these two series of data (temperatures 1259° and 1646° Abs.; for the latter the data is insufficient to draw a good curve) the values of A were computed by the present system, equation (6). This gave $A = 2890$ and $A = 2920$, respectively, and a mean value of $C = 14\,430$. However, since so few observations were at hand to draw the curves, and since the energy curve at the lower temperature fitted best the Wein computation for λ_m, and since the energy curve for 1646° seemed to fit neither the Wein nor the Planck system for computing λ_m, no great reliance is placed on this computation.

To the writer it seems that all this data should be considered from the standpoint of historical interest, since it seems impossible to give it much weight in connection with the results obtainable at the present day. All the work done in recent years is found to be at variance with the uncorrected results of Paschen, and of Lummer and Pringsheim. The first intimation of this fact came from a research by Holborn and Valentiner[22] in the visible spectrum.

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[21] Lummer and Jahnke: Ann. der Phys. (4), 3, p. 283, 1900.

[22] Holborn and Valentiner: Ann. der Phys. (4), 22, p. 1, 1906.

They found a value of $C = 14\ 200$. Recently Valentiner[23] has corrected this value for lack of blackness of the radiator, arbitrarily raising the value to $C = 14\ 350$.

The investigations by Warburg[24] and his assistants is not yet published in complete form. The preliminary data for isochromatic curves gives a value of $C = 14\ 380$ and for isothermals $C = 14\ 360$. In the latter the range is from $C = 14\ 230$ to 14 480. Considered as a whole, a fair estimate of the results of the foregoing investigations indicate a value of $C = 14\ 400$ to 14 460.

IV. APPARATUS AND METHODS

Under this title are given only the main outlines of the instruments used in this research. For a more complete description it is necessary to consult the original papers to which frequent reference is made.

1. Bolometer and Galvanometer.—For the first part of this investigation an air bolometer was used, which was placed at the focus of a hemispherical glass mirror, 5 cm in diameter and having an opening 0.5 by 1.5 cm at the center. The most of the observations were made with a vacuum bolometer, known as bolometer No. 10, placed at the focus of the hemispherical glass mirror just mentioned. The sensitive platinum strips of the bolometer were 10 by 0.6 by 0.0003 mm, the resistance of each strip being 5.6 ohms. The resistance of the complete bolometer was 8.9 ohms (depending of course upon the battery current and the vacuum) with a temperature coefficient of 0.023 ohms per degree. The galvanometer resistance was 5.09 ohms at 20° C, with a temperature coefficient of 0.0194 ohms per degree. It was usually operated on a complete period of three to five seconds, and sensitivity of 2 by 10^{-10} ampere.

The rear side of each bolometer strip was bright, and the front side was painted with a mixture of lampblack and platinum black, then smoked with soot from a paraffin candle. The fluorite window was covered with thin sheet copper, which was painted with lampblack, and which formed a knife-edge slit, from 2 to 3 mm in width, for admitting upon the bolometer the part of the spectrum which was to be measured. The rear wall and tube leading to the viewing window of the glass vacuum chamber was painted with lampblack to absorb or diffuse any radiation which passed by the bolometer strip. A more detailed description of the radiometric apparatus has already been published,[25] and for clearness the vacuum bolometer is illustrated only by a simple diagram in Fig. 1.

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[23] Valentiner: Ann. der Phys. (4), **39**, p. 489, 1912.

[24] Warburg, Leithauser, Hupka, and Müller: Zs. Instk. **32**, p. 134, 1912. As this paper goes to press a recent communication (Sitzber Akad. Wiss. Berlin, **I**, p. 35, 1913) gives what appears to be a final value of $C = 14\ 374 \pm 40$ and $A = 2894 \pm 8$.

The reflecting power of lampblack and of platinum black have also been published.[26] The results show that the reflecting power of lampblack and of platinum black is low and fairly uniform through the spectrum. The amount lost by diffuse reflection from the bolometer would be returned upon its path owing to the presence of the hemispherical mirror.

A large glass bottle of about 4 liter capacity prevented a noticeable change in the gas pressure, resulting from leakage, and more especially from a change in vapor pressure with temperature of the material used in cementing the joints of the vacuum bolometer. Since a quite accurate knowledge of the bolometer resistance is required, the gas pressure was reduced to only about 0.2 mm (cathode dark space 3 to 4 mm) at which pressure the bolometer resistance is still quite independent of the degree of evacuation.

The sensitivity of the bolometer-galvanometer was varied by means of a resistance box R_2 (see wiring diagram, Fig. 1) placed in series with the galvanometer. By this means the galvanometer deflections (maximum allowed was 12 to 14 cm) were kept within the range of proportionality with the current flowing through the galvanometer. In previous papers[26a] it was shown that the galvanometer deflections were proportional to the current for deflections up to 20 cm; also that the galvanometer deflections were proportional to the energy falling upon the bolometer strip for intensities far exceeding those to which the bolometer was exposed in the present research. This is of importance in connection with the question of the constants, observed at high temperatures.

A standard resistance of 100 000 ohms was kept in parallel with one branch of the bolometer, see Fig. 1, S_1. By short-circuiting 20 000 ohms of this resistance the current in the bolometer circuit

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[25] This Bulletin, **4**, p. 392, 1908; **9**, p. 7, 1912 (vacuum bolometer).

[26] This Bulletin, **9**, p. 283, 1913.

[26a] See note 25, p. 17.

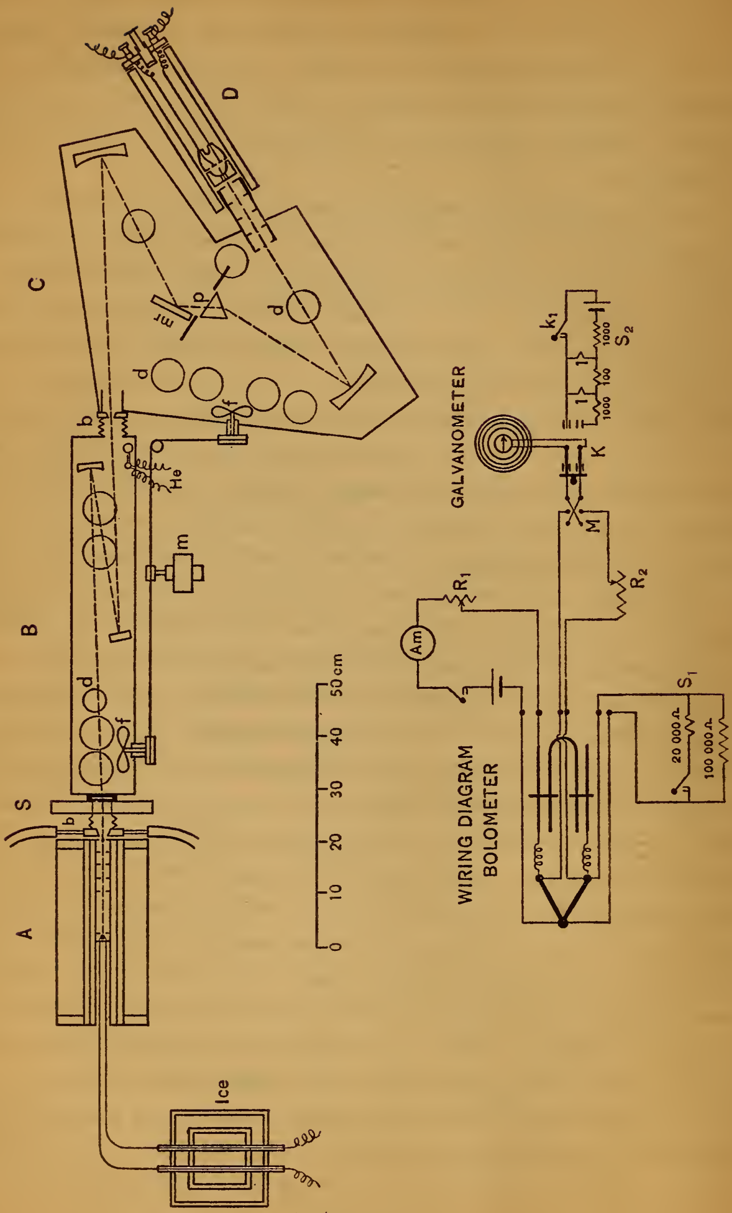
A
S
B
C
D
b
b
d
d
d
d
f
f
m
He
p
m_1
Ice
0
10
20
30
40
50 cm
GALVANOMETER
WIRING DIAGRAM
BOLOMETER
Am
R_1
R_2
M
K
k_1
S_1
S_2
20 000 Ω
100 000 Ω
1000
100
1000
1
1

Fig. 1

was unbalanced, causing a galvanometer deflection which was used as an arbitrary standard for testing the electrical sensitivity of the bolometer-galvanometer circuit. The galvanometer sensitivity was tested by throwing the switch K (Fig. 1) to the right and closing the switch K_1, through the battery of S_2.

No standard was provided for testing the radiation sensitivity (which varied with the gas pressure) owing to the difficulty in proving a standard radiator which could be easily operated in the spectrometer. The main test of the radiation sensitivity of the instruments was a repetition of the observations which had been made at the beginning and during the series of observations. The bolometer was usually operated on a constant current of 0.03 ampere, and it was usually found that if the galvanometer sensitivity remained constant the radiometric observations could usually be repeated to 1 part in 200. After exhausting the air the tendency of the radiation sensitivity is to slowly decrease. The electrical test of sensitivity using the standard resistance S_1 was, of course, frequently applied while observing a spectral-energy curve. Usually the galvanometer sensitivity[27] would remain constant throughout a series of observations on an energy curve, which required from one to one and one-half hours. During the first year's work, however, when an air bolometer was used, it required from two to three hours to observe a complete energy curve, and much difficulty was experienced from variation in sensitivity of the galvanometer, which required a complete period of six to seven seconds in order to attain a sufficiently high sensitivity. At that time from 15 to 20 galvanometer readings, at any given spectrometer setting, was required to obtain a reliable mean value. With the bolometer in vacuo the same precision was attained in 5 to 6 readings of the galvanometer, which, owing to the high radiation sensitivity of the bolometer, could be operated on a complete period of four to five seconds. Usually about 30 spectrometer settings and from 200 to 250 galvanometer readings (including the repetitions) were required to obtain a complete energy curve. Hence, a conservative estimate of the total number of deflections recorded during the past four years is in excess of 40 000 observations.

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[27] In the afternoon the galvanometer sensitivity would increase 1 or 2 per cent and return to its original intensity the following morning, owing probably to the diurnal variation of the earth's magnetic field.

The present apparatus is the result of experiments extending over at least five years, during which time the radiation instruments have undergone repeated alterations and have been tried out on various subsidiary problems. The energy curves are not perfectly satisfactory, however, and, as is mentioned elsewhere, they probably never will be until the complete optical path is inclosed in a vacuum to eliminate atmospheric absorption bands.

In observing galvanometer deflections there is constantly a fluctuation in the readings amounting to 0.1 to 0.5 mm, whether or not there is a resistance in the galvanometer circuit for reducing the large deflections obtained when exposing the bolometer to intense radiation. The precision is therefore usually no higher for high than for low intensities, i. e., for large (reduced by resistance when being observed) or for small deflections. The proper method when using high intensities is to compensate the deflections by means of some sort of potentiometer device and reducing the operation to a null method, or at least to compensate for the greater part of what would be the true galvanometer deflection and read only the residual deflection. In this manner the aforementioned errors would be only in the residual deflection. However, such a device may contain other inherent errors, and in view of the great variation in the intensities measured, it did not seem justifiable for this part of the research to depart from the well-tried bolometric method of observing the direct galvanometer deflection. When operating the radiator above 1100° it was an easy matter to reduce the aforementioned fluctuations in the galvanometer readings by reducing its sensitivity; and with a galvanometer (complete) period of only four to five seconds a precision of 1 part in 200 to 300 could easily be attained. It is, therefore, an open question whether at these temperatures a greater precision could have been attained by null methods. In fact, considering the numerous other difficulties and the time saved, it is doubtful whether anything would have been gained by using other than the direct galvanometer deflection method of observation.

2. **Spectrometer.**—The spectrometer, from Fuess, was remodeled to suit the present investigation. The mirrors were 10 cm in diameter and 52 cm in focal length. An optically plane mirror 10 by 5 by 0.8 cm was mounted upon an especially constructed prism table. With the prism it formed the well-known Wadsworth method for maintaining minimum deviation.

The spectrometer circle was divided to 10′ with a glass vernier, which read directly to 1′. The micrometers for measuring angles to less than 1′ were never used; for the bolometer was purposely constructed to subtend 4′ of arc (the spectrometer slit and bolometer strips were 0.6 mm wide) and the spectrometer settings were made to the even minutes of arc. This was done to simplify the reduction of the data to the normal spectrum, before any of the observations were plotted for computing the constants.

The mirrors were silvered by the Brashear process and polished with the finest optician's rouge. The whole optical system, Fig. 1, was inclosed in a box of thin sheet copper, with three openings in the top, which gave access for removing and adjusting the mirrors. Although the silver mirrors could be kept an indefinitely long time in this box without tarnishing,[28] they were silvered at the beginning of each year's work.

The lids covering these openings were closed with soft wax. Within the box, in line with, but below the optical path, were numerous large glass dishes *d*, *d*, containing fresh phosphorous pentoxide, while other vessels contained freshly broken sticks of caustic potash. The vessels not below the optical path were wrapped in black velvet. Small fans, *f*, *f*, Fig. 1, driven by a motor, *m*, were operated, before starting a series of observations, in order to dry the air.

The experimental work was planned for the winter months when the humidity was low. This was as important as the air stirring and the drying devices. The water vapor can be reduced to a fairly low value with drying material; but the carbon dioxide can not be thoroughly eliminated. There is also evidence that the absorption bands of oxygen[29] at 3.2μ and 4.7μ may cause quite an appreciable depression in the energy curves. Lummer and Pringsheim (loc. cit.) mentioned such an absorption band but did not know what substance caused it. It may have been caused by hydrocarbon vapors; but owing to the long optical path (about 2.5 m) from the radiator to the bolometer it seems more probable that atmospheric oxygen exerts an appreciable absorption.

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[28] This Bulletin, 7, p. 197; 1911 ("Preservation of Silver Mirrors").

[29] Investigations of Infra-red Spectra, Carnegie Publication, No. 35, p. 49; 1905. Subsequent observers have not found these bands in oxygen.

The entire interior of the spectrometer box was, of course, painted with lampblack in a dilute alcoholic solution of shellac, to absorb scattered light.

The spectrometer arms were set so that the rays fell upon the collimating mirrors at as small an angle of incidence as was possible, when in connection with the bolometer. The astigmatism was therefore very small, and it was not increased a measurable amount by increasing the angle of incidence upon the mirror, which occurs in rotating the prism table in order to project the infra-red spectrum upon the bolometer strip. This might be of importance when using a slit image which covers the whole length of the bolometer strip, as was done in some of the present work; but by actual measurement it was found that this possible source of error, if present at all, was negligible.

3. **Prisms and their Adjustment.**—The greater part of the work was done with a flawless fluorite prism (No. 1) having a refracting angle of 60° and circular faces 33 mm in diameter. Although this prism seemed to belong to the green variety of fluorite it was practically colorless and entirely free from internal cleavage planes. An examination of the transmission of samples of light green fluorites [30], which contained appreciable coloring matter showed that it is highly improbable that this prism has any absorption bands, which, if present, should occur at 1.5μ and 3μ. The possible absorption bands of fluorite would be extremely weak, and they would occur (e. g., the 3μ band) in the region of atmospheric absorption bands, where no computations were made. There was also available a large prism, No. II (refracting angle 60°; faces 52 by 50 mm) which belonged to the purple variety of fluorites. It was colorless but contained numerous internal cleavage plates which scattered much light. It will be shown that, probably as a result of this scattering, only about 25 to 30

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[30] This Bulletin, 9, p. 116; 1912.

per cent of the observed spectral energy curves fitted the Planck equation, when about 80 of the energy curves produces by the small prism, operated under similar conditions, fitted the Planck equation. The other prisms used in the latest work will be described in a forthcoming paper. The face of the prism *p*, Fig. 1, was covered with a diaphragm of cardboard, painted with lampblack, which prevented radiations (other than those entering the prism) from passing beyond the central part of the spectrometer box.

All the adjustments were made with the prism mounted upon its table. This includes the adjustment of the prism to the vertical axis of the spectrometer, the adjustment of the prism faces to parallelism with the auxiliary mirror *mr*, Fig. 1, the measurement of the angle between the prism face and the mirror; and the angular setting necessary to cause the light to pass through the prism at minimum deviation.

The method is fully described elsewhere[31] and is too lengthy to repeat here. In order that all the rays may pass through the prism at minimum deviation, the angle of incidence must vary as the rays fall upon the auxiliary silver mirror *mr*, Fig. 1. The angle between the mirror and the adjacent prism face is about 80°. On a subsequent page it will be shown that the correction for variation of the reflecting power with angle of incidence is negligible.

The alignment of the radiator and the adjustment of the mirrors in the box *B*, Fig. 1, so that in projecting an image of the radiating wall *d*, Fig. 2, upon the spectrometer slit the spot of light passes centrally through the prism *p*, Fig. 1, is extremely delicate, and at the beginning of each new series of observations (as well as during the observation of an extensive series of energy curves) the spot of light coming from the radiator, heated to 1250 to 1300°, was adjusted to pass accurately through the central part of the prism. The importance of this is evident from the fact that the spot of light was about 27 mm. in diameter while the prism was only 33 mm. in diameter, so that any slight lateral displacement of the image falling upon the spectrometer slit was liable to throw the light partly off the prism face. After the light had passed through the prism the spectrum was adjusted upon the bolometer strip by means of the adjusting screws which are attached to the intervening mirror.

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[31] Investigations of Infra-red Spectra, Carnegie Publication No. 35, p. 18; 1905.

4. **Radiator or so-called Black Body.**—The unmounted porcelain tubes *A*, *B*, *C*, in Fig. 2 were purchased abroad, which explains much of the delay in concluding this investigation, owing to the difficulty in promptly obtaining the proper material. The outer porcelain tube *C*, Fig. 2, was surrounded with a ring of asbestos cloth, and the intervening space was filled with fine magnesium carbonate, which, however, on heating was partly decomposed into the oxide and had to be refilled several times during the first part of the work. The ends were of kaolin.

The outer heating tube *B* was 5.7 cm in diameter. It was wound with a strip of platinum 0.02 mm in thickness and 1 cm wide, and after some usage was fastened permanently within the tube *C* by means of kaolin. This part of the radiator was used throughout the work. To aid in temperature compensation the platinum ribbon was closely wound at the ends with only a few turns in the center.

The inner tube *A*, which was 3 cm in (internal) diameter, had to be removed frequently in order to repair the platinum winding, to be replaced by another tube when broken and to facilitate adjustments, etc. The ends rested upon porcelain rings in order to keep the proper adjustments, and when once in place for a series of observations the open spaces between the tubes *A* and *B* were closed with a thick paste of kaolin, sectioned radially to allow for expansion, shrinkage, etc. This tube was uniformly and closely wound with a strip of platinum 0.01 mm in thickness. The distance between the successive turns of platinum ribbon was of the order of 0.3 to 0.5 mm, so that the porcelain was almost entirely covered with metal. The width of the strip was 2 cm at the center and decreased uniformly to 1 cm width at the ends as shown in Fig. 2. It required about 220 cm of platinum ribbon cut in this manner to cover the inner tube.

In the earlier part of the work attempts were made to regulate the temperature uniformity by varying the heating current in the two coils, *A* and *B*, by means of separate rheostats in the circuit with the two windings of these two porcelain tubes. It was found,

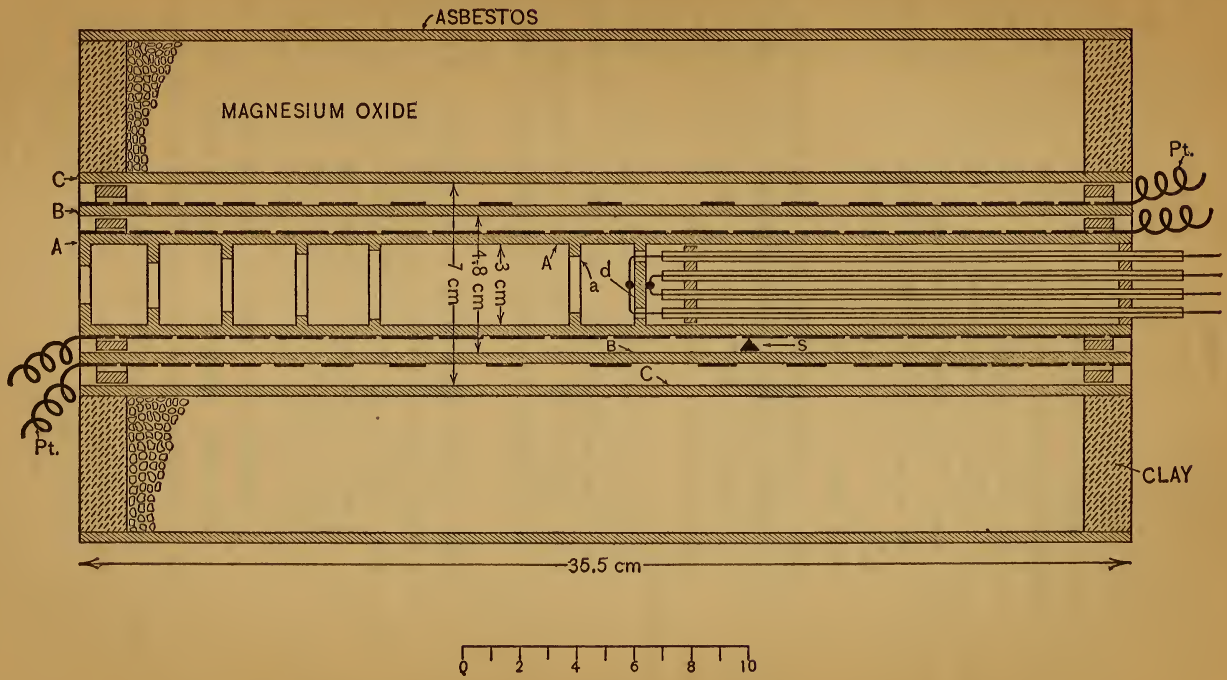
ASBESTOS
MAGNESIUM OXIDE
C
B
A
Pt.
Pt.
7 cm
4.8 cm
3 cm
A
a
d
B
C
S
CLAY
35.5 cm
0
2
4
6
8
10

Fig. 2

however, that at the high temperatures used, the heating by radiation was far more effective than heat conduction. It was found more satisfactory (and less laborious) to join the two heating circuits in parallel with the main line, which contained the rheostats, and allow each tube to take its proportionate part of the current than to attempt to regulate the current in each radiator by means of separate rheostats. Hence, at some temperatures the radiators were at a uniform temperature over a greater length of the tubes than at a different temperature. However, it will be shown presently that the temperature was uniform over a far greater length than required by theory so that variations in the length of the radiator over which there was temperature uniformity is of minor importance.

The use of platinum ribbon instead of thick platinum wires eliminates the question of local nonuniformity of temperature. The central 2.5 cm length of tube was so wound that one turn of the platinum ribbon entirely covered what comprises the theoretical black body.

On several occasions the heating coil of the inner tube broke during a series of observations, and the radiator was heated by radiation from the outer coil, *B;* but the computations of the energy curves do not indicate a systematic difference in the values of the constants.

The construction of the apparatus, e. g. position and size of opening in the water-cooled shutters, was such that no radiation from the side walls, which may be hotter than the radiating diaphragm, *d*, Fig. 2, could be projected upon the spectrometer slit. Furthermore, the diffuse reflecting power,[32] especially that of the radiators painted with cobalt or chromium oxide, is sufficiently low so that only an imperceptible amount of radiations from the hotter side walls, which fall upon the radiating diaphragm, *d*, will be reflected upon the optical path leading to the spectrometer slit. In measuring the total radiation from such an inclosure, the extra amount of radiation, which (as a result of diffuse reflection of the radiations from the hotter side walls) is superposed upon the radiations from the diaphragm under investigation, might be measurable. But after dispersing these radiations into a spectrum it would be a hopeless task to attempt to detect the increase in emissivity, at any spectral region, which might be caused by the superposition of diffusely reflected radiations from the side walls upon the radiations emitted by the diaphragm.

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[32] This Bulletin, 9, p. 283, 1913.

The radiating diaphragm was 30 mm in diameter, but the arrangement of the collimating mirrors, *B*, Fig. 1, was such that the image of only about 18 mm of this diaphragm was projected upon the spectrometer slit. The spectrometer slit was 10 mm in height, so that only the radiations from about 10 mm of the central portion of the radiating diaphragm entered the spectrometer. It was only by this arrangement that consistent results were obtainable.

TABLE 3

Showing the Drop in Temperature Within the Radiator at Different Distances from the Radiating Wall; a Rise in Temperature is Indicated by a Plus Sign

Distance from radiating wall; cm	0 to 0.5	1	2	3	4	5	6	8	10	12
Temp. centigrade of radiating wall	Drop in temperature									
I 612°	0°	0	0	....	0	....	0.5	4.3	13.5	....
980	0.3?	0	0	....	+1.5	....	+0.8	4.0	....	....
1225	0	....	0	....	0	....	0	6.2	18	....
1275	0	....	....	0.1	....	....	0	6.0	....	....
1283	0	....	....	....	....	....	0.6	1.2	....	....
1300	0	....	0.5	....	1	....	2	7	20	....
II 910°	0	....	+0.6	+0.5	0	....	1	4	....	....
975	+0.6	....	+1.7	....	+1.3	....	0	3	....	....
1147	+0.6	+1.1	+0.8	....	....	1.3	2.2	6	9	....
III (Marquardt) 950°	0	....	0	....	....	0	....	....	0?	16

In the first work, "Series of 1909," the radiator was directly before the slit, as used by Lummer and Pringsheim. This, of course, reduced the length of the optical path and hence atmospheric absorption, but it was impossible to be certain that no radiations other than those from the diaphragm could enter the spectrometer slit. This was especially true for the first part of this series when following the custom of previous investigators; no attempt was made to prevent the inner tube, *A*, Fig. 2, of the radiator from sagging, which occurs when heated to 1200°. The fragment of porcelain, with a knife-edge, *s*, Fig. 2, about 2 mm long, was used to prevent the inner tube from sagging. It was placed about 2.5 to 3 cm to the rear of the radiating diaphragm. No local cooling could be detected as the result of its presence. This support was one of the chief factors in the successful operation of the radiator. In fact the success attained in the present research is owing to three things, (1) the introduction of a support to prevent sagging, (2) more thorough insulation of the thermocouples by enlarging the holes in the radiating wall to cover the thermocouples with thicker-walled porcelain tubing; this (3) permitted the use of blackened radiators.

The length of porcelain tube between the radiating wall and the first perforated diaphragm, *a*, Fig. 2, was about 2.5 cm. After the experience gained in winding the first radiators, there was no difficulty in spacing the compensation coil and cutting the platinum ribbon of the inner heating coil so that, at all temperatures, there was a uniformity in temperature to 0.5° for a distance of 6 cm or more from the radiating diaphragm and a drop in temperature of only 10 to 18° at a distance of 10 cm in front of the radiating wall. In the second "Series of 1909," after rewinding the furnace, the temperature of the air, at a distance of about 2 cm from the radiating wall, was about 1.5 to 1.8° higher than the temperature of the diaphragm.

In these tests for temperature uniformity within the radiator it is to be remembered that the experiments were made with (1) the exploring thermocouple drawn through a small hole in the radiating diaphrahm, and (2) by inserting the thermocouple, inclosed in thin porcelain tubes, through the opening in the front of the radiator. In using the latter method there was evidence showing that the drop in temperature was partly owing to a cooling of the radiator by the removal of the thermocouple. Series III of Table 3 was made on a Marquardt porcelain tube by method (1) using a thermocouple of wires 0.1 mm in diameter, and the temperature was found uniform for a distance of 12 cm from the radiating wall, where it suddenly dropped 16°. The small fluctuations in temperature uniformity may be due, in part, to lack of homogeneity in the exploring thermocouple.

From the fact that the radiator was heated to a uniformity of 1°, over three times the length of the theoretically perfect radiator, it appears as though a uniformly heated inclosure had been attained to a sufficiently high degree of accuracy for the work at hand. By cutting the platinum wire much narrower at the ends the temperature compensation could have been produced over a greater length of the radiator, but there appeared no advantage radiometrically, by so doing. On the other hand, it would have been disadvantageous in quickly heating the radiator to have the platinum cut much narrower at the ends; for, owing to the low heat conductivity of the porcelain, it would become overheated at the ends when starting, and the tube would break or its outer surface would crack. As used, the temperature of the radiator could be raised to 1500° in less than three hours without danger of breaking the porcelain tube. It was found, however, that, after a tube had been heated 25 or 30 times, the platinum seemed to shrink into the porcelain, causing a pinkish colored compound which, on account of its different expansion coefficient, would chip loose and, by warping, would tear the platinum ribbon.

The radiating diaphragm was adjusted and focussed upon the spectrometer slit by inserting into, and illuminating, the interior of the inclosure by means of an especially prepared Nernst glower, 1 cm long, which was operated on a transformer. In this manner, especially when using unblackened tubes, there was no difficulty in making these adjustments. On interchanging radiators, tube *A*, Fig. 2, and before making any adjustments, or after making a series of observations, the image of the center of the thermocouple was always found to be within 1 mm of the center of the slit. The water-cooled shutter was then secured in place and the adjustments tested. This glower was, of course, removed before making observations. The radiator had, of course, to be adjusted so that the light fell centrally upon the prism face. This was accomplished by heating the radiator to 1200° to 1300° and adjusting the optical bench upon which it rested. The adjustments were also tested by sending the light from a Nernst glower through the spectrometer in the reverse direction. For this purpose the viewing microscope was removed from the bolometer, *D*, Fig. 1, and the light from a Nernst glower was passed along its axis. This of course produced a spectrum, part of which passed through the spectrometer slit and from thence passed out into the radiator. With a right-angled prism or mirror it was possible to determine the accuracy of the adjustments upon the diaphragm; or with the inner tube,

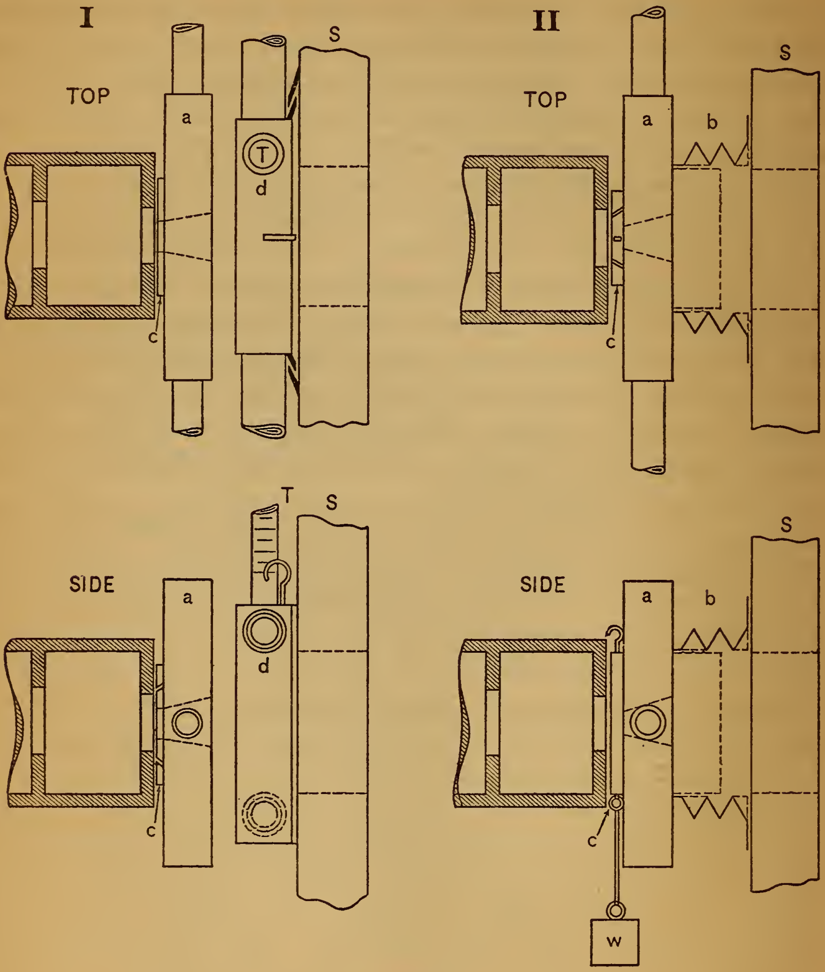

Fig. 3

A, Fig. 2, removed it was possible to adjust the axis of the radiator. These various adjustments were preliminaries, and as already mentioned the final adjustment was made by heating the radiator to a high temperature and tracing the spot of light emanating therefrom through the entire apparatus.

5. **Water-Cooled Shutters.**—Two different types of water-cooled shutters were used in this investigation. The first type, "shutter No. 1," shown in Fig. 3, I, consisted of a large stationary copper shield, *S*, 1 cm in thickness and 5 by 50 cm in area which was filled with water. To this shield, was attached a shutter, *d*, of thin sheet copper, 5.5 by 6 in area and 1 cm in thickness, which could be raised or lowered in front of the opening in the shield, *S*, thus admitting radiation into the spectrometer. This shutter contained a thermometer, *T*, and through it a stream of water was kept flowing. The temperature of the water flowing in the shutter, *d*, varied from 6 to 15°, depending upon the temperature of the radiator and the outside air. Water was also kept flowing through the shield, *a*, which was 6 cm in diameter and 1 cm in thickness. This shield contained a slot into which was slipped thin brass plates, *c*, having openings varying from 3 to 6 mm. After the black body radiator had been adjusted to the spectrometer, the shield, *a*, was clamped firmly in front of the opening in the radiator. This outfit was used on "Series of 1909" where the black body stood directly in front of the spectrometer slit. The shutter, *d*, was also used with a similar shield *S*, of somewhat smaller dimensions, Fig. 1, in "Series IX, 1912." However, it permitted too much (humid) air to enter the spectrometer case, hence it was discarded. The second type of shutter, Fig. 3, II, consisted of a stationary shield, *S*, of thin copper, 40 by 20 and 1 cm thickness, shown at *S* in Fig. 1, which was filled with water. Water was kept flowing through the smaller shield, *a*, which was 6 in diameter and 1 cm in (internal) thickness. After the radiator was adjusted, this shield, *a*, was centered before the opening and rigidly clamped at a distance of 2 or 3 mm from the end of the radiator. As a further precaution this shield was fastened to the end of the black body radiator by means of plaster of Paris. The shutter, *c*, consisted of a sheet of brass about 1.2 mm in thickness and 2 by 1 cm in area which moved in vertical ways close against the water-cooled shield, *a*. It therefore had very closely the temperature of the shield, which as in shield No. I varied from 6 to 15° C. The shutter, *c*, was raised by means of a cord and pulley, as shown in the photographic illustration, Fig. 5, accompanying this paper, and, owing to the friction in the slots

required a small weight, *w*, to be attached permanently to cause it to return, by gravity. The knife-edge hole, *d*, in the diaphragm, *a*, was 4 mm in diameter, which, by the geometrical construction of the apparatus, did not permit radiations from the side walls to fall upon the spectrometer slit. The two shields, *S*, and *a*, were connected with a flexible leather bellows, *b*, which prevented air from passing into the spectrometer box. It avoided also jarring and displacement of the adjustments by expansions and contractions in the radiator. The shutter, *a*, was placed as close to the radiator as the expansion of the porcelain tube would permit, and the intervening space was packed with asbestos. By opening this shutter only when observations were made, it was possible to reduce the humidity of the inclosed air in the spectrometer box and keep it fairly constant while making observations. No attempt was made to keep a record of the temperature of the thin metal shutter, *a*, for it never differed sufficiently from the bolometer to enable one to detect experimentally radiations passing to or from the bolometer. Any corrections that would have to be made to the spectral energy curves, owing to an exchange of radiations between the shutter and the bolometer, would fall beyond 6μ, while the computations of the present research usually do not extend beyond 5.8μ.

6. **Temperature Scale and Temperature Measurements.**—The temperature of the radiator or so-called black body was determined by means of two Heraeus thermocouples of *Pt* and *Pt* + 10 per cent *Rh*. One of the thermocouples was placed horizontally across and in contact with the front side of the radiating diaphragm of the black body, as shown in Fig. 4, *A*, which is a section viewed from above. The lead-wires passed out through the rear of the radiator to the ice box, Fig. 1. In the "Series of 1912" when the inner walls and the front side of the radiating diaphragm were painted with chromium oxide or cobalt oxide, which becomes electrically conducting at 1200°, the front thermocouple was completely inclosed in a porcelain insulating tube, as shown in Fig. 4, *B*. The short piece of porcelain which lies across the radiating wall of the black body was covered with cobalt oxide. Since the cobalt oxide did not adhere well to the porcelain tube, the latter was first covered with a thin layer of iron oxide, obtained by wetting the

porcelain tube with writing ink which was burned into the tube. The cobalt oxide paint was then applied and also burned upon the tube by means of a blast lamp. After replacing the thermocouple inclosed in this short tube within the radiator the whole interior was again painted with a mixture of cobalt and chromium oxides. In order to be able to insert the thermocouple including the short tube, and also to paint the interior of the radiator, the first diaphragm (which in the commercial porcelain tube has an opening of only 1 cm) was removed, Fig. 4, *B*, and after making all the adjustments, another porcelain diaphragm was put in its place.

The cobalt oxide has a high temperature coefficient of absorption, so that it appears black on slight heating. Its emission spectrum [33] is continuous so that there is less difficulty in producing a perfect radiator than when the radiating inclosure is of white porcelain.[34] However, porcelains having a low melting point when heated above 1000° emit a far more continuous spectrum than the "Marquardt porcelain" from which are made the black bodies ordinarily used.

All these tubes, especially the painted ones, become electrically conducting when heated above 1300°, so that for higher temperatures the radiator was heated by alternating current from a motor-generator which was operated from a storage battery. By this means there was no more difficulty in maintaining a constant current than when heating the radiator directly by means of current from a storage battery. By means of a low resistance rheostat at the observing table, the current could be regulated so that the temperature of the radiator could be kept constant to a few hundredths of a degree. No observations were made, especially at high temperatures, when the temperature fluctuated as much as 0.1° from the mean value at which the radiator was being operated.

The temperature of the front side of the radiating wall was regulated by the heating current and the temperature of the rear side of the radiating wall was brought to that of the front side by changing the size of the opening, Fig. 4, at the rear end of the radiator. This was done by means of asbestos fiber. The amount of adjustment of the opening had, of course, to be regulated at every heating of the radiator, and also during a series of observa-

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[33] Investigations of the Infra-Red Spectra, No. 97, p. 120, Carnegie Institution of Washington, 1908.

[34] This Bulletin, 9, p. 81; 1912. (See p. 106.)

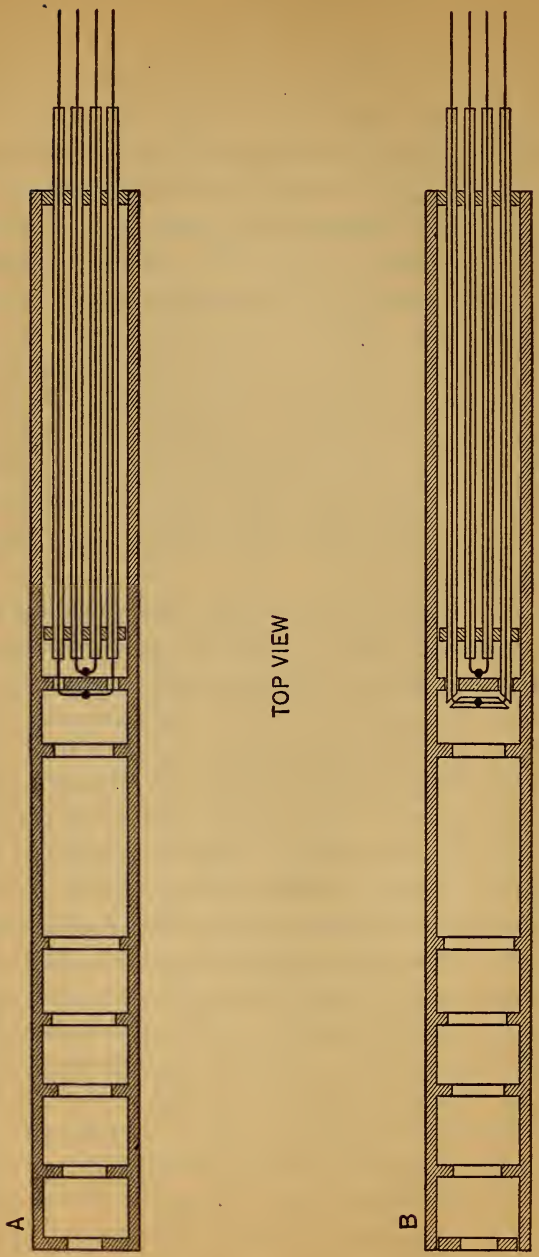

Fig. 4

tions. This was entirely a matter of experience and there was no difficulty in keeping the temperature difference of the two sides of the radiating diaphragm to within a few tenths of a degree. It was then assumed that the true temperature of the radiating diaphragm was that measured by the thermocouples; and it was felt that the temperature measurements were as well known as is the temperature scale. It is to be noticed, however, that the temperatures used fall well within the region of the known temperature scale. An error of 1° or 2° in the measurements of the temperature would mean an accuracy of 1 to 2 parts in 1400 to 1800, which is a far greater precision than is possible with the other measurements involved in the experiments.

The thermoelectric measurements were made by means of a potentiometer and a standard cell, both of which were standardized by the Electrical Division of this Bureau at the beginning of a series of measurements.

The thermocouples were frequently standardized by the Heat Division of this Bureau, by determining their emfs at the freezing points of zinc (419.°2), antimony (630.°0), and copper (1083°). From these emfs and temperatures the calibration curve was computed by the formula

$$E = a + bt + ct^2$$

This is the thermoelectric scale which on extrapolation beyond 1400° differs from the optical scale [35] based on the Wien equation using $C = 14\,500$ (i. e., differs from the thermodynamic scale) by the amounts given in Table 4.

TABLE 4

Temperature on thermoelectric scale	1200 C	1300	1400	1500	1600°
Optical—Thermoelectric (mean values)	0	2	6	14	25
Optical—Thermoelectric used by W. W. C.	0	0	1	7	18

The temperature scale used in the present investigation, for temperatures above 1400°, is the Optical Scale [36] which is obtained by adding to the thermoelectric temperatures a proportionate amount of the corrections given in the lower line in Table 4. This correction is somewhat smaller than given by Waidner and Burgess, owing to the fact that their corrections relate to the mean values of five different thermocouples. The emfs of the thermocouples used in the present research correspond very closely with the thermocouple, P_3 (used by Waidner and Burgess), which indicated temperatures 5 to 7° higher than the mean value. Hence the corrections (which become operative at 1400° C.) to reduce the observed temperatures to the optical scale are smaller by 5 to 7°, depending, of course, upon the temperature. The correction is quite large at 1500°, which may account for some of the high values of the constant, C, which still appear to be a trifle larger at high temperatures than at low temperatures. However, this occurrence of high values of C at high temperatures is not sufficiently systematic for the observations on different years to conclude that it is owing to this correction to the temperature scale (i. e., thermocouple readings reduced to the gas scale) or to a deviation of the observed curve from the Planck equation, but is the result of other experimental errors. In future work it is purposed to calibrate the thermocouples by using the melting point of palladium as the highest fixed point, thus eliminating this correction. This correction is of minor importance since but few energy curves were obtained at temperatures higher than 1400° C., and hence can not affect the mean value of the present results. The correction has no effect whatever upon the data obtained in 1912 (which is the crucial work), and ordinarily it might have gone unmentioned.

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[35] Waidner and Burgess; this Bulletin, 3, p. 205, 1907, Table 20.

[36] Modified as explained in the text; for, using the unmodified factors, the values of $\lambda_m T$ show an abnormal and systmatic increase in value at temperatures higher than 1400° C. We are therefore forced to choose between the possibility (1) that this variation is owing to the failure of Planck's law at high temperatures or (2) that, for the particular thermocouples used in this research, the given correction factors for reduction to the optical scale are too large. The writer has chosen the latter conclusion, and the result-obtained at high and low temperatures are now in agreement.

7. **Description of the Assembled Apparatus.**—In Fig. 5 is shown a photograph of the radiometric apparatus as used in the latter part of the investigation. The essential parts of the apparatus are shown in Fig. 1. Beginning at the extreme left of the plate, the double-walled ice box, *I*, contained the cold junctions of the thermocouples. The radiator is at *A*, and is shown in more detail in Fig. 2. An inner platinum-wound tube (broken) is shown at *Pt*. The main rheostats are at *R*, and the ammeter is shown at *Am*. The water-cooled shutter and the water-cooled diaphragm are shown at *S*, Figs. 1 and 4.

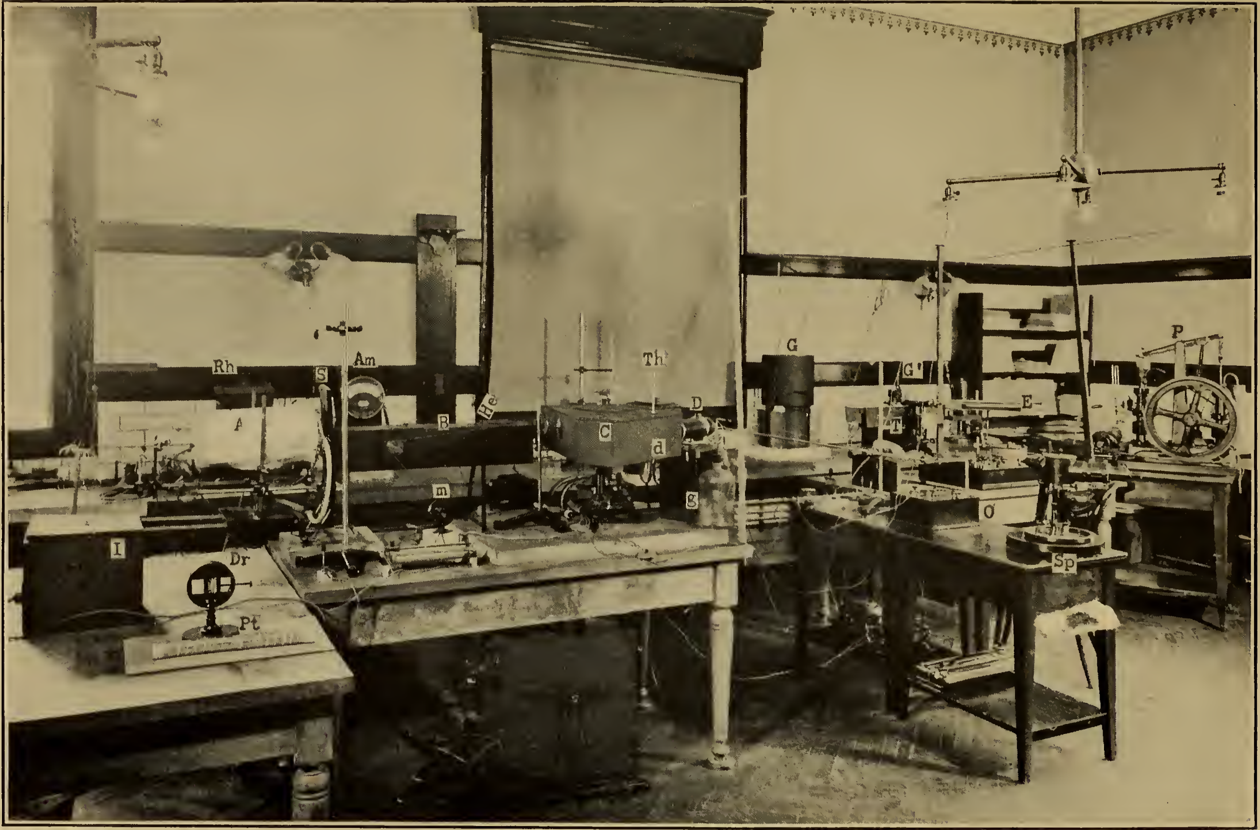

Fig. 5.—*Radiometric laboratory.*

The air-tight metal box, *B*, contains a plane mirror and a concave mirror (5 cm in diameter, 50-cm focal length) for focusing the radiating diaphragm (*d*, Fig. 2) upon the spectrometer slit. It contains also glass dishes with dry potassium hydroxide and phosphorus pentoxide. The helium tube, *He*, can be rotated about a vertical axis so that it comes in front of the spectrometer slit. The tube is excited by a 2000-volt transformer situated at *t*, and the air is agitated by fans (*f*, *f*, Fig. 1) operated by a motor, *m*.

The spectrometer, with its optical parts inclosed in an air-tight metal box, is shown at *C*. The glass dishes containing phosphorus pentoxide are below the optical path and in the metal boxes, *d*, the bottoms of which may be unscrewed to renew the phosphorus pentoxide. The three lids on top of the spectrometer box admit adjustment of the optical parts. They are closed with universal wax. A thermometer is shown at *Th*. The spectrometer circle is viewed by means of a telescope, *T*, and is rotated by a lever, both of which are situated on the small observing table, *O*. The spectrometer and the radiator are joined to the box, *B*, by means of flexible leather bellows (*b*, *b*, Fig. 1), which prevents any strain acting upon the spectrometer arm.

The vacuum bolometer is within the metal case, *D*. For clearness of illustration the Geryk oil pump, used in evacuating the bolometer, was placed at *P*. When in operation it is joined to the large glass bottle, *g*, which is in series with the bolometer. The bolometer is kept balanced by rotating the light metal arms which extend to the observing tables, *O*. The auxiliary galvanometer inclosed in its iron shields is shown at *G*. It is mounted upon a massive wall-bracket upon which is placed a heavy slab of marble.

Upon the large observing table, *O*, are situated a telescope and scale, *T*, for reading the galvanometer, *G*; a potentiometer (the auxiliary moving coil galvanometer is at *G′*) and illuminated scale, *E*; and mercury switches leading to the thermocouple wires (shown strung across the room to the ice box, I). Upon the small observing stand are mercury and knife switches which are accessory to the bolometer and its galvanometer; also a telescope for setting the spectrometer circle and a dial resistance in series with the galvanometer, *G*. On the lower shelf of this table is shown the standard 100 000-ohm box, and a low resistance, *r*, which is used in regulating the electric current used in heating the radiator, *A*, and maintaining it at a constant temperature. The water-cooled shutter is opened and closed by means of a cord which passes over the pulley at *S*. By means of these devices the observer, at *O*, was able to control all the operations necessary in order to obtain a spectral energy curve. These operations included testing of the galvanometer-bolometer sensitivity, balancing the bolometer, setting the spectrometer circle, maintaining the radiator at a constant temperature, reading the galvanometer deflections, etc.

The mounting for the hemispherical mirror used in an investigation of diffuse reflection [37] is shown at *Dr*. The short-focus spectrometer used in photographing the light of the firefly,[38] and for obtaining spectral energy curves [39] in the visible and ultra-violet parts of the spectrum is shown at *Sp*.

The multiple sectored disk [40] (without its motor) previously described is shown at *Sd*.

8. **Refractive Indices Used for Calibration.**—Until within the past four years there was a wide discrepancy in the indices of refraction of rock salt and of fluorite, as determined in Europe and in this country. The foreign investigators using mirror spectrometers of about 35 cm focal length and bolometers subtending an angle of 3 to 6′ of arc attempted to attain an accuracy which they supposed far outweighed the precision attainable by Langley and Abbot with their spectrobolometer, which had a focal length of 2.3 meters and subtended an angle of 7″ to 1′.3. In this particular kind of work "size is a most important element of accuracy." [41]

The explanation of this discrepancy was finally obtained in 1908, when Paschen [42] published a redetermination of the indices of refraction of rock salt, which are in perfect agreement with the values obtained by Langley. This gave a clue to the discrepancy in the refractive indices of fluorite. Langley and Abbot had found the refractive indices of fluorite indirectly by comparing bolographs of the solar spectrum which had been obtained by means of the two kinds of prisms. Therefore Paschen's refractive indices of flourite should agree with those obtained by Langley and Abbot. Applying the correction for temperature coefficient [43] to Paschen's most recent determinations of the refractive indices of fluorite,[44] which are considered the most reliable, it was found that his values fall exactly upon the dispersion curve of fluorite published by Langley.[45] Hence, for the region of the spectrum to 3.5μ to which Langley's work applies, the curve (Pl. XXVI *B*, loc. cit.) of refractive indices, which represents the observations of both observers, was adopted in the present research. This is a large plate, and the

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[37] This Bulletin, **9**, p. 283, 1913, J. Franklin Inst., **1**, 74, p. 549; 1912.

[38] This bulletin, **7**, p. 243; 1911.

[39] Publication No. 164, Carnegie Institution of Washington, 1912.

[40] This bulletin, **7**, p. 243; 1911.

[41] Langley: Annals Astrophys. Obs., vol. 1, p. 222; 1902.

[42] Paschen: Ann. der Phys. (4), **26**, p. 120; 1908.

TABLE 5

Refractive Indices, n, of Fluorite at 20°; Data from Langley and from Paschen

Wave length	n	Log n	Wave length	n	Log n
0. 5893	1. 43390	0. 1565180	2. 6519	1. 42019	. 1523455
. 6562	257	. 1561158	2. 700	. 41988	. 1522517
. 6867	200	. 1559430	2. 800	923	. 1520528
. 71836	157	. 1558126	2. 850	890	. 1519518
. 76040	101	. 1556426	2. 9466	823	. 1517467
. 8840	1. 42981	. 1552783	3. 050	1. 41748	. 1515169
1. 000	896	. 1550200	3. 098	714	. 1514114
1. 100	834	. 1548316	3. 241	600	. 1510900
1. 1786	789	. 1546948	3. 400	487	. 1507134
1. 250	752	. 1545822	3. 5359	376	. 1503788
1. 3756	1. 42689	. 1543905	3. 8306	1. 41119	. 1495855
1. 4733	642	. 1542474	4. 000	. 40963	. 1491051
1. 5715	596	. 1541073	4. 1252	847	. 1487476
1. 650	558	. 1539916	4. 250	722	. 1483620
1. 7680	502	. 1538210	4. 400	563	. 1478710
1. 840	1. 42468	. 1537173	4. 600	1. 40355	. 1472279
1. 900	439	. 1536274	4. 7146	233	. 1468502
1. 9153	431	. 1536046	4. 800	130	. 1465311
1. 9644	407	. 1535313	5. 000	1. 39900	. 1458177
2. 0626	357	. 1533789	5. 3036	9522	. 1446427
2. 1608	1. 42306	. 1532232	5. 8932	1. 38712	. 1421141
2. 250	258	. 1530758	6. 4825	7821	. 1393154
2. 3573	198	. 1528920	7. 0718	6805	. 1361020
2. 450	144	. 1527272	7. 6612	5670	. 1324838
2. 5537	080	. 1525329	8. 2505	1. 34440	. 1285290

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[43] Micheli: Ann. der Phys. (4), **7**, p. 772; 1902. Liebreich: Verh. Phys. Gesell., **13**, p. 1; 1911.

[44] Paschen: Ann. der Phys. (4), **4**, p. 302; 1901.

[45] Langley: Annals Astrophys. Obs., Smithsonian Institution, vol. 1, p. 222, Plate XXVI, B; 1902.

refractive indices are drawn to a scale of 1 mm = 0.002μ. Beyond 3.5μ Paschen's refractive indices were employed. The values adopted in computing the calibration curve used in the present research are given in Table 5. The necessity of this high accuracy arises in computing the slit-width correction for reducing the observations from the prismatic to the normal spectrum.

The value of the refractive index for λ = 0.58932μ was adopted from a consideration of the observations of Stefan, Sarasin, Pulfirich, Simon, Carvallo, Martens, and Paschen. The values of these various observers differ but little from the adopted value, the greatest difference amounting to perhaps 5″ in the minimum deviation of the D-lines of sodium.

That high precision was attained in the present calibration is evident from the fact that, by a single measurement, the intense emission line of helium[46] was located at λ = 1.084μ. The mean value of a large number of determinations by Paschen[47] using a large grating is λ = 1.083μ.

A variation in temperature of 1° causes a change in refraction of about 1″.75 in a fluorite prism. The experiments of Micheli and Liebreich (loc. cit.) show that the temperature coefficient of refraction is quite constant for the infra-red region of the spectrum concerned in the present investigation, so that the correction for change in dispersion is negligible. The essential requirement is to maintain an accurate setting of the bolometer upon the sodium lines, the "zero setting."

The calibration was computed for 20° C, and since the temperature within the spectrometer box departed only about 1° from this value, no correction is necessary to the observed data.

9. **Fiducial Reference Lines.**—The position which the bolometer occupies in the spectrum is determined from the calibration curve computed from the minimum deviation settings on the spectrometer circle, and the corresponding wave lengths. These minimum deviation settings are computed from a knowledge of the angle of the prism and the values of the refractive indices just discussed. The fiducial reference line, the "zero" of the spectrometer circle, is the sodium line λ = 0.5893μ. For convenience this "zero" of the spectrometer circle is set at some whole number, say 10° 0′

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[46] This Bulletin, **9**, p. 81, 1912. (See p. 95.)

[47] Paschen: Ann. der Phys. (4), **27**, p. 537, 1908.

o″, and the sodium line is caused to fall upon the bolometer strip. If the temperature changes appreciably during the observations the refraction of the prism changes noticeably and the sodium line will no longer fall upon the bolometer strip when the "zero setting" is at 10°. This was the method used for checking the adjustment at the beginning and end of every series of observations. The bolometer was provided with a micrometer screw[48] for making this adjustment.

The use of the sodium lines, $\lambda = 0.5893\mu$ as a standard of reference was inconvenient, and the more intense yellow helium line $\lambda = 0.5876\mu$ was adopted for the "zero setting." Repeated tests during the past five years failed to indicate an error as great as 5″ in setting the bolometer on these lines. The helium tube was pivoted so that it could be swung in front of the spectrometer slit, as shown in Figs. 1 and 5.

10. **Corrections for Selective Reflection.**—The radiations from the black body are reflected five times at silvered surfaces and must pass through the fluorite prism and the fluorite window covering the bolometer. Experiments made by Paschen and by the writer show that the transmission of white fluorite is uniform throughout the spectrum to about 7μ, so that there is no correction for absorption in fluorite. In this research there are three corrections to the spectral energy curves: (1) A correction for variation in the reflecting power (R_1) of the fluorite surfaces (four in all, R^4_1), which is a function of the wave length and the refractive index; (2) a correction for the variation in reflecting power (R_2) with variation in angle of incidence upon the prism face (two reflections, R^2_2); and (3) a correction for the variation in the reflecting power (five reflections, R^5) of silver with wave length.

The value of R_1 was computed by the well-known Fresnel formula; $R_1 = [(n-1) \div (n+1)]^2$ using the refractive indices given in Table 5. The values are given in Table 6. The correction for variation in the reflecting power (R_2) with angle of incidence upon the prism face was computed from the Fresnel formula:

$$R_2 = \frac{I}{I_0} = \frac{1}{2}\,\frac{\sin^2(i-r)}{\sin^2(i+r)} + \frac{1}{2}\,\frac{\text{tang}^2(i-r)}{\text{tang}^2(i+r)}$$

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[48] See Fig. 9, this Bulletin, 4, p. 439; 1907.

where i = angle of incidence and r is the angle of refraction. For minimum deviation $r_1 = r_2 = \alpha/2$ where α is the refracting angle of the prism.

The correction for the variation in the reflecting power of silver, with variation in the angle of incidence, was investigated [49] and found to be only 0.12 per cent per degree of angular rotation for angles of incidence between 54° and 64°. Since the total angular rotation in passing from 1μ to 6μ is less than 4° this correction is negligible. As already mentioned the whole optical system being inclosed and the moisture eliminated, the silver mirrors could be kept indefinitely. However, they were silvered each year, at the beginning of a new series of observations.

TABLE 6

Showing the Variation in Reflecting Power, R_1, of Fluorite with Refractive Index, n; and R_2, with Variation in the Angle, i, of Incidence

λ	$R_1 = \left(\frac{n-1}{n+1}\right)^2$	$(100-R_1)^4$	i		R_2	$(100-R_2)^2$	$(100-R_1)^4$ $(100-R_2)^2$
μ		Per cent	°	′		Per cent	Per cent
0.360	0.0332	87.37	46	16.7	0.0442	91.36	79.80
.397	.0328	.52		8.7	.0436	.47	80.07
.486	.0322	.72	45	56.3	.0426	.66	.40
.589	.0317	.91		48.3	.0420	.78	.66
.760	.0314	88.02		41.2	.0414	.89	.90
.884	.0313	.06		38.3	.0412	.93	.98
1.179	.0311	.13		33.5	.0409	.99	81.04
.473	.0309	.21		30.0	.0406	92.04	.19
.572	.0308	.23		28.8	.0405	.06	.24
.768	.0307	.27		26.5	.0404	.08	.28
2.063	.0305	.34		23.0	.0401	.14	.36
.357	.0304	.38		19.0	.0398	.20	.50
.947	.0300	.53		10.0	.0392	.31	.79
3.241	.0297	.64		4.7	.0388	.39	.92
.536	.0294	.76	44	59.0	.0384	.47	82.04
4.125	.0288	.96		46.2	.0374	.66	.40
.715	.0281	89.23		31.4	.0364	.85	.80
5.304	.0273	.51		14.2	.0352	93.08	83.36
.893	.0263	.89	43	55.0	.0338	.35	.97
6.482	.0253	90.25		33.7	.0324	.62	84.44
7.072	.0242	.67		9.7	.0308	.93	85.19
.661	.0229	91.15	42	43.2	.0291	94.26	.93
8.250	.0216	.64		14.5	.0272	.59	86.74

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[49] This Bulletin, 9, p. 283, 1913.

The values for the reflecting power of silver were adopted after a consideration of the experimental data of various experimenters, especially the data of Hagen and Rubens, and of Paschen. The data are given in Table 7. Beyond 1.7μ the reflecting power of silver is practically constant and for convenience in computation the last column in Table 7 gives the correction for five reflections on the basis that beyond 2μ the reflecting power of silver is constant. A new determination of the reflecting power of silver [50] by a slight modification of the method used for measuring the diffuse reflecting power of matte surfaces gave values which are in excellent agreement with the results of other, more elaborate methods used in determining reflectivity.

TABLE 7

Showing the Reflecting Power of Silver for 3 and for 5 Mirrors

λ	R	R^3	R^5	R^5 (2 to 8μ= 100%)	λ	R	R^3	R^5	R^5 (2 to 8μ= 100%)
μ					μ				
0.40	0.840	0.593	0.418	0.465	0.90	0.960	0.885	0.815	0.906
.45	.880	.681	.528	.587	1.00	.964	.895	.833	.926
.48	.894	.714	....	....	1.20	.972	.918	.868	.966
.50	.900	.729	.590	.660	1.30	.974	.924	.877	.976
.55	.914	.763	....	....	1.40	.976	.928	.883	.982
.60	.925	.791	.678	.754	1.52	.977	.933	.890	.990
.65	.935	.817	....	....	1.607	.978	.935	.895	.996
.70	.942	.835	.741	.824	1.800	.9785	.937	.897	.998
.75	.947	.849	....	....	2.027	.979	.938	.899	1.000
.80	.951	.857	.774	.861	2.043	.979	.938	.899	1.000

From this it will be noticed that provided the optical system is enclosed, which preserves the silver mirrors, these corrections need be computed but once, and it is felt that they are just as reliable as (if not more reliable than) direct experimental determinations of the absorption in the apparatus which would require an additional spectroradiometric outfit. Such a direct experimental determination of the absorption in the apparatus is made daily by Mr. Abbot at the Astrophysical Observatory; but here the silver mirrors are out in the open and the measurements relate to the visible and the ultra-violet spectrum, where the optical properties of silver are entirely different from those that obtain beyond 1μ in the infra-red. By actual measurement, while engaged in the determination of the reflecting power of metals,[51] the increase in absorption of a silver mirror (which to the eye appeared slightly tarnished from standing in the air) could not be detected with certainty beyond 1.5μ when compared with an untarnished piece of this same mirror which in the meantime had been preserved in a desiccator as a standard of reference. In the ultra-violet, of course, this tarnishing would have caused a very appreciable absorption. As already stated, the spectrometer mirrors being entirely enclosed did not become tarnished [52] at least not during the few months during which a series of observations were in progress. Moreover, the application of this correction decreases the final numerical values by only about 5 parts in 1000. The correction may be even less than this amount, for this estimate was obtained by recomputation of some of the older observations in which these correction factors had not been used. It was then found, on recomputing the observations obtained at low temperatures and on redrawing the curves (from which the constants are eventually obtained) that the ordinary errors of observation were sufficient to mask the effect of the correction. The writer can not, therefore, consider as well taken any criticism that may be raised in regard to the elimination of this possible source of error, for there are too many other errors of far greater magnitude.

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[50] This Bulletin, 9, p. 283, 1913.

11. **Methods of Reduction of Data to Normal Spectrum.**—As already mentioned, the spectrometer settings—i. e., the calibration of spectrum—is obtained by computing the minimum deviation settings from the angle of the prism and the refractive indices given in Table 5. These settings are usually, of course, uneven numbers so that it is better to adopt an even number (say, 10° 0′ 0″) on the spectrometer circle. For example the minimum deviation setting for the sodium lines is 31° 36′ 31″. The minimum deviation setting for $\lambda = 1.7680\mu$ is 30° 52′ 53″. The difference is 0° 43′ 38″, and since the prism table must be rotated through one-half the required angle, the bolometer strip will be at $\lambda = 1.7680\mu$ when the prism table is rotated 0° 21′ 39″ from the zero position (the

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[51] This Bulletin, 7, p. 197; 1911.

[52] See "Note II, Preservation of silver mirrors," this Bulletin, 7, p. 221; 1910.

sodium lines at 10° 0′ 0″). Of course, in practice the spectrometer settings were made on the exact divisions of the spectrometer circle, the corresponding wave lengths being read from the calibration curve, and the values tabulated, Table 8. For example, with the sodium lines on the bolometer and the spectrometer circle reading 10° 0′ 0″ a rotation of 1° 0′ 0″ (spectrometer reading 9° 0′ 0″) placed the bolometer strip at $\lambda = 4.0307\mu$ in the spectrum.

The calibration curve was plotted to a large scale to insure accuracy. The scale adopted was 1 cm. = 30″ = 0.02μ or 1 mm. = 3″ = 0.002μ. The cross-section paper used was printed from an extremely accurate lithographic plate, 75 by 75 cm. area. This large plate made it possible to plot from 8 to 12 points in one continuous line thus increasing the accuracy. Owing to the uncertainty in the values of the refractive indices (by Paschen, Rubens, Carvallo, Langley) available when the radiation work was first begun, this calibration curve has been computed at least five times during the past seven years. The greatest uncertainty lies in the region of 1.7μ where the dispersion of flourite has a point of inflection. This is manifested in the "slit width" correction which reduces the prismatic to the normal spectrum. The method of obtaining this correction will now be described, and it is shown in Fig. 6. From the calibration curve, the wave lengths, corresponding to every minute on the spectrometer circle, were tabulated. The bolometer was constructed to subtend 4′ (width = 0.6 mm) and from the tabulated data just mentioned it was possible to determine the wave lengths comprised in 4′ of arc, in different parts of the spectrum. These values of $d\lambda$ were then plotted as ordinates against the wave lengths to which they correspond in the spectrum, as shown in Fig. 6. The earlier computations gave a wavy series of points which fall regularly above and below the smooth curve. This is no doubt due to systematic errors in the older values of the refractive indices. The final values computed from the refractive indices given in Table 5 fall close to the curve. However, in the region of 1.7μ, where there is a point of inflection, it was necessary to adjust the values to a smooth curve. The uncertainty in the values of $d\lambda/d\theta$ in this region of the spectrum fall in the fourth decimal place and while the variations appear large, it is to be

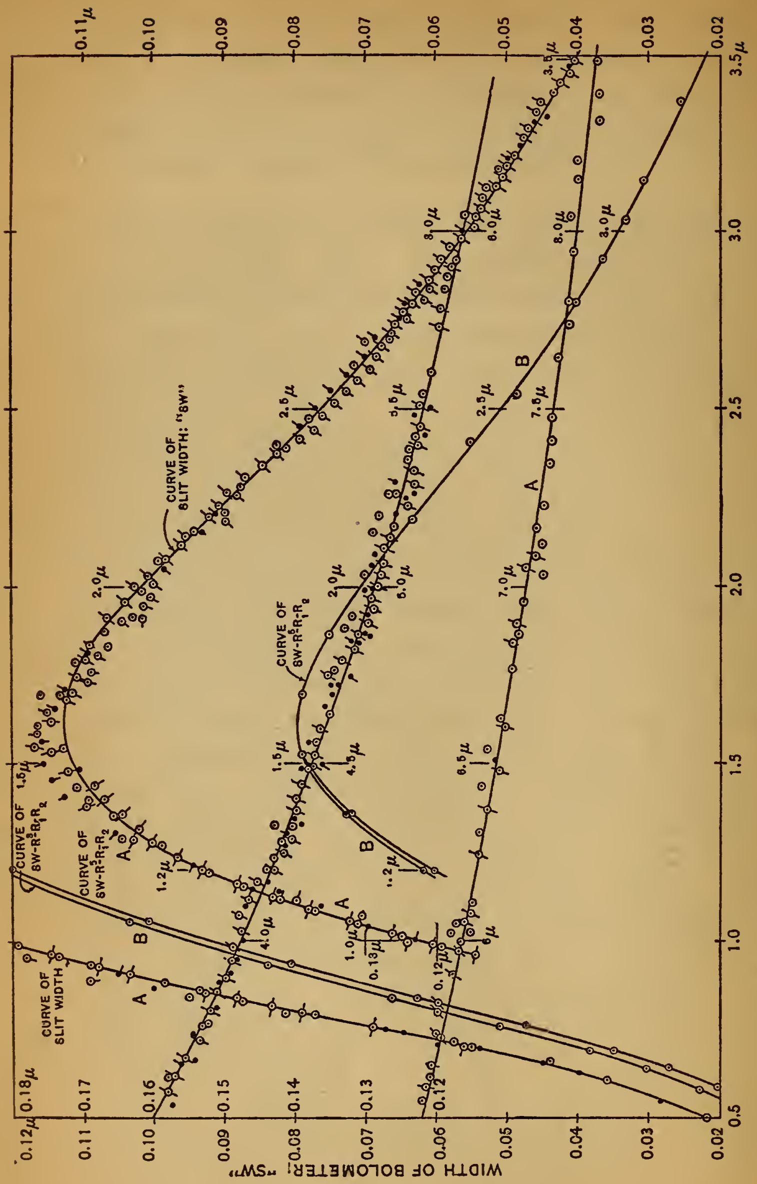

Fig. 6.—*A. Showing the width of the bolometer in different parts of the spectrum*

B. Complete correction curve for reducing date from prismatic to normal spectrum, when using 3 or 5 silver mirrors, and 4 fluorite reflecting surfaces. See Table VIII noticed that in comparison with the whole range of the spectrum they are quite small. This "slit width" curve is an excellent check upon the accuracy of the calibration curve.

The values of the "slit widths" *SW* (read from the original curve, of which Fig. 6 broken into several parts is a copy) of the bolometer for reducing from the prismatic to the normal spectrum are given in column 3 of Table 8. Columns 4 and 5 of Table 8 give the complete factors for reducing the present observations to the normal spectrum. They include the corrections for selective reflection at the prism faces, fluorite window, mirrors, etc., as given in Tables 6 and 7.

In practice the spectral energy curves were observed at every 2′ on the spectrometer circle which corresponds (by the Wadsworth minimum deviation system) to 4′, the width of the bolometer strip. From these observations the second term factor $(-\frac{1}{6}f(\alpha))$ which is necessary for reducing to a "pure spectrum" could be determined directly, the complete formula[53] for reduction to a pure spectrum being:

$$\begin{cases} D'_0 = D_{0'} - \frac{1}{6}\left(\dfrac{D_{-2'} + D_{+2'}}{2} - D_{0'}\right) + \ldots . \\ D_n = D'_{0'} \div sw. \end{cases}$$

In this equation $D_{0'}$ is the observed galvanometer deflection at any spectrometer setting, o'; $D_{-2'}$ is the observed deflection at a point $-2'$ preceding this setting; and $D_{+2'}$ is the deflection at a setting $+2'$ in advance; and $D'_{0'}$ is the true reading for a "pure spectrum." In order to reduce these values to the normal spectrum, D_n, the values of $D_{0'}$ are then divided by the "slit width" (s. w.) correction factors given in Table 8 using column 6 when five mirrors were employed in the spectrometer system. These values of D_n (reduced to the normal spectrum) were then plotted upon accurately made cross-section paper (1 cm = 0.2μ) and a smooth curve, from which were read certain wave lengths (as will be explained presently) for computing the position, λ_m, of the maximum emission, E_m, of the curve. From this it will be seen that the determinations of the radiation constants involve graphical as well as mathematical computations.

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[53] Runge: Zs. für Math. u. Phys., **42**, p. 205; 1897. Paschen: Ann. der Phys. (3) **60**, p. 714; 1897.

TABLE 8

Spectrometer settings		Wave lengths in μ=0.001 mm	Width of bolometer (0.6 mm=4′) in wave lengths; the "slit-width, s w."	Complete slit width correction; prismatic to normal spectrum, 3 mirrors	Complete slit width correction; prismatic to normal spectrum, 5 mirrors
°	′				
	12	0.4491	....	....	....
	10	.4660	0.0173	....	....
	8	.4855	.0195	....	....
	6	.5058	.0216	....	....
	5	.5171	.0228	....	....
	4	.5298	.0248	....	....
	2	.5579	.0283	....	....
10	0	.5893	.0338	0.0229	0.0203
	58	.6250	.0395	.0275	.0245
	57	.6462	.0430	.0303	.0271
	55	.6975	.0530	.0390	.0351
	53	.7567	.0690	.0510	.0470
	52	.7940	.0788	.0585	.0545
	51	.8355	.0880	.0664	.0626
	50	.8820	.0983	.0747	.0710
	49	.9338	.1088	.0836	.0804
	48	.9908	.1203	.0935	.0903
	47	1.0541	.1322	.1033	.1007
	46	1.1230	.1430	.1129	.1107
	45	1.1971	.1530	.1215	.1198
	44	1.2760	.1603	.1280	.1267
	43	1.3571	.1650	.1328	.1317
	42	1.4410	.1692	.1365	.1358
	41	1.5262	.1721	.1390	.1386
9	40	1.6132	.1718	.1397	.1395
	39	1.6980	.1714	.1390	.1388
	38	1.7835	.1700	.1372	.1372
	37	1.8685	.1660	.1354	.1354
	36	1.9518	.1630	.1328	Values are the same as for a 3-mirror system
	35	2.0330	.1590	.1300	
	34	2.1128	.1555	.1288	
	33	2.1895	.1515	.1235	
	32	2.2654	.1480	.1204	
	31	2.3385	.1444	.1182	
9	30	2.4098	.1414	.1149	
	29	2.4791	.1368	.1117	
	28	2.5458	.1332	.1090	
	27	2.6120	.1302	.1068	
	26	2.6757	.1275	.1043	
	25	2.7392	.1253	.1024	
	24	2.8010	.1227	.1003	
	23	2.8618	.1203	.0985	
	22	2.9213	.1183	.0966	
9	20	3.0373	.1140	.0932	
	18	3.1501	.1109	.0907	

TABLE 8—Continued

Spectrometer settings	Wave lengths in μ=0.001 mm	Width of bolometer (0.6 mm=4′) in wave lengths; the "slit-width, s w."	Complete slit width correction; prismatic to normal spectrum, 3 mirrors	Complete slit width correction; prismatic to normal spectrum, 5 mirrors
16	3.2600	0.1075	0.0878	Values are the same as for a 3-mirror system
15	3.3132	.1060	.0865	
14	3.3670	.1045	.0852	
12	3.4700	.1007	.0828	
10	3.5690	.0980	.0806	
8	3.6665	.0954	.0787	
6	3.7600	.0930	.0766	
4	3.8520	.0912	.0750	
3	3.8970	.0903	.0743	
2	3.9423	.0893	.0735	
9 00	4.0307	.0873	.0718	
55	4.2438	.0825	.0681	
50	4.4455	.0785	.0649	
45	4.6385	.0750	.0621	
40	4.8272	.0716	.0594	
35	5.0010	.0687	.0571	
8 30	5.1685	.0663	.0552	
25	5.3290	.0641	.0534	
20	5.4855	.0623	.0520	
15	5.6410	.0604	.0505	
10	5.7900	.0588	.0492	
5	5.9380	.0573	.0481	
8 0	6.0770	.0556	.0469	
7 55	6.213	.0544	.0457	
54	6.240	.0542	.0456	
50	6.348	.0530	.0447	
45	6.4775	.0520	.0439	
40	6.6055	.0505	.0429	
35	6.7200	.0495	.0420	
7 30	6.853	.0487	.0413	
20	7.090	.0468	.0401	
10	7.326	.0450	.0387	
7 00	7.546	.0433	.0378	
50	7.760	.0420	.0375	
40	7.966	.0407	...	
6 30	8.170	.0395	...	
20	8.363	.0375	...	
10	8.550	.0360	...	
6 00	8.737	...	...	
50	8.918	...	...	
40	9.09	...	...	
5 30	9.2670	...	...	

From the factors given in Table 8 it may be noticed that the reduced values of the observations, in the short wave lengths, are suppressed and that the values in the long wave lengths are greatly magnified in comparison with the original observations. This decreases the accuracy of the value of λ_m when computed from values of λ_1 and λ_2 situated far apart. Fortunately this is the region of the spectrum where errors in λ_2 have but little effect upon the computation of λ_m. For values of λ_1 and λ_2 near together the corrections for selective reflection, "slit width," etc., are closely of the same value, so that the errors introduced are much reduced. Unfortunately the presence of atmospheric absorption bands prevented extensive computations at the top of the energy curve, where λ_1 and λ_2 are close together and where the various factors are of the same magnitude on both sides of the energy curve. After eliminating atmospheric absorption, the method of obtaining the constant, C, by taking the values of λ_1 and λ_2 close together on an isothermal energy curve should not be much inferior to the method of isochromatics, owing to the small difference in the correction factors on the two sides of the maximum.

12. **Computation of λ_{max}.**—The computation of the location of the maximum emission is made from two wave lengths, λ_1 and λ_2, which have the same emissivity, $E_{\lambda1} = E_{\lambda2}$ on the spectral energy curve, using the formula equation (6) given on a preceding page. No computations were made in regions of the spectrum where λ_1 or λ_2 coincided with atmospheric absorption bands. In some respects this is a wasteful procedure, since the region of 2.5 to 3.2 μ, 4.1 to 4.3 μ, and 5.8 to 6.2 μ can not be utilized. However, for the series of 1911, which was determined when the humidity was exceptionally low, the water vapor has but little effect in the region of 2.9 to 3.0 μ. In addition to the absorption bands, the energy curves may be defective in other regions of the spectrum. However, there usually was no difficulty in obtaining from 8 to 10 values of λ_1 (and λ_2), which were widely separated on the energy curve. As already mentioned, the computation is partly graphical and for this purpose the energy curve was plotted upon an accurately ruled plate having an area of 40 by 50 cm, ruled in millimeters.

The computation of λ_m from values of λ_2 falling in the region of 5.9 to 6.1 μ, always gave low values, which was assumed to be due to atmospheric absorption bands of water.[54] The computation of λ_m for λ_2 falling in the region of 6.0 μ was therefore not used in making up the mean value of λ_m.

Attention has already been called to the fact that when using the first term of equation (6) for computing λ_m the values show a uniform variation. This is well illustrated in Fig. 7, A, which is obtained from a theoretical (Planck) curve. Fig. 7, B are typical experimental curves. The line slanting to the horizontal axis shows the computation when using the first term of equation (6), which is the same as equation (1), with the additional factor α/α'. This shows that the Wien equation is not applicable. The horizontal line through the mean value, resulting from computing λ_m by using all of equation (6) aids in illustrating the departure of the individual values from the mean value of λ_m. The abscissas

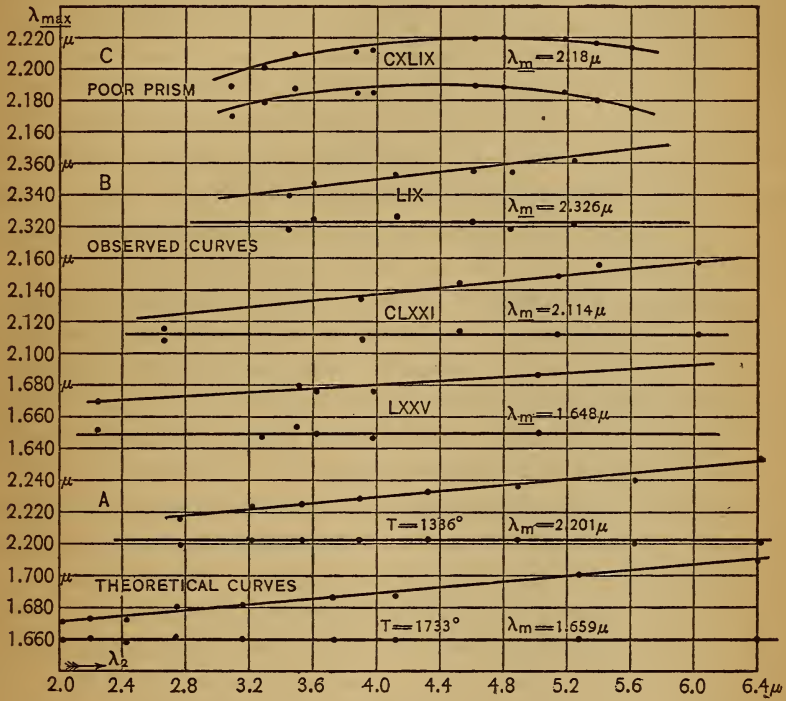

Fig. 7

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[54] This Bulletin, 7, p. 619; 1911.

are the location of the values of λ_2. Of course, instead of λ_2 the corresponding values of λ_1 might have been plotted. It may be noticed that the wide departures from the mean usually occurs when λ_2 falls in a region of atmospheric absorption at 3 μ. Energy curve No. LXXV is of interest because it gives uniform values of λ_m, but a high value of A, which seems to indicate an error in the temperature.

The curve of λ_m, Fig. 7, C, was obtained with the large fluorite prism, as already described. It fits neither the Planck nor the Wien curve, owing to an elevation at 3.5 to 4.5 μ in the energy curve. As will be noticed presently, 75 to 80 per cent of the observed energy curves fit the Planck curve, as interpreted from the closeness with which the individual computations fall to the horizontal line representing the mean value; the tolerance being $\pm 0.015\ \mu$ for curves having the maximum emission in the region up to 2.5 μ and $\pm 0.02\ \mu$ for λ_m lying in the region of 3 μ.

13. **Accuracy Attainable.**—It is a difficult matter to give an estimate of the accuracy attainable in the final results owing to the many variable factors connected with the problem. The temperature measurements are no doubt accurate to 1 part in 1500, which is entirely beyond the range of accuracy attainable in radiation measurements at the present day.

In radiation measurements at any one spectrometer setting, an accuracy of 1 part in 200 to 400 could easily be attained. But using different radiators operated at the same temperature (to 1°), as was usually done, or operating the same radiator at the same temperature on different days did not, as a usual thing, produce energy curves which were in agreement to better than 1 per cent throughout the whole spectrum. This is, of course, owing to changes in sensitivity of the radiometer, to variation in humidity, etc. It is a severe test that has heretofore never been undertaken.

The computation of the constants involves some uncertainties, but owing to the large-sized and accurately ruled cross-section paper used this is of minor importance. The scale to which the energy curves were plotted was 1 mm $= 0.02\mu$, and it was an easy matter to read the values of λ_1 and λ_2 to 0.005 μ. Frequently when the curves were poor the values were read to 0.01 μ. It was the common experience to find that for values of λ_1 and λ_2, ranging over the whole curve, the individual values of λ_m differed by 0.005 to 0.01 μ which is of the order of 1 part in 300 to 200 since the λ_m falls between 2 and 3 μ. This is owing to the fact that small variations in the value of λ_2 (which fall in the region of the spectrum where the energy is weak and difficult to observe) have but little effect in equation (6). It is for this reason that the present method seems superior to the determination of isochromatic energy curves, in spite of the fact that by the latter method the absorption in the apparatus is eliminated. By taking λ_1 and λ_2 over a wide range of the spectrum, the errors in calibration (in refraction and in the slit-width correction), in changes in sensitivity, etc., are reduced to a negligible value. This is owing to the fact that when E_1 and E_2 (for λ_1 and λ_2) are close together they fall in the doubtful region at 1.6 to 2 μ (for the temperatures investigated), where the slit-width reduction factors are known with the least accuracy; but they are of nearly the same value for $E_1 = E_2$. When λ_1 and λ_2 are far apart, quire a large error (owing to the uncertainty in the correction for reflecting power at λ_1) must occur before it becomes noticeable in computing λ_m.

There appears to be no systematic variation in the different values of the constant, *C*, with temperature (which might result from errors in the temperature scale and in the computation of λ_m), from which it appears that the Planck formula is sufficiently accurate for the temperature from 500° to 1500°. The best observational work could be done in the range of 800° to 1400°, the lower temperature being such as to give a sharp energy curve and the higher temperature being still within the range of knowledge of the temperature scale.

The effect of "field light" scattered over the spectrum owing to the imperfections in the prism, was tested in various ways, the assumption being made that this scattered light was uniformly distributed, which of course could hardly be true for diffuse reflection. One test for diffuse light was to set the bolometer at 0.32 μ, where the reflecting power of silver is practically zero, and measure the spectral radiation of a highly incandescent body, e. g., a Nernst glower. For the perfectly clear fluorite prism used in this work no radiation could be detected. Another test was to set the bolometer in the yellow part of the spectrum, at a point where a sample of Schott's monochromatic red glass becomes opaque, the source of radiation being a Nernst glower. The test was made on a very windy day, when a high precision was not possible, but for measurement to 1 part in 300 no scattered radiations could be detected. That the amount of diffuse light is extremely small in this prism (No. 1) is shown indirectly from the data on the transmission of water and of cupric chloride solutions (this Bulletin, **7**, p. 619, and **9**, p. 110), which are entirely opaque in the region of 1.2 to 2 μ, where one would expect stray radiation.

These tests were not made on the large fluorite prism, described elsewhere, except at 0.32 μ, where the amount of diffuse light was barely measurable. Nevertheless, to the eye there appeared to be considerable diffuse red light extending far into the infra-red. Furthermore the spectral energy curves obtained with this prism are distorted by an excess of radiation in the region of 3.5 to 4 μ, which appears to be caused by diffuse radiations superposed upon the spectrum. It will be shown presently that but very few (25 per cent) of the spectral energy curves obtained with this prism fit the Planck equation, and the constant, C, is much higher than obtains with any of the determinations with the perfectly clear prism. Perhaps this will explain a similar condition in the curves obtained by Paschen (loc. cit.) and his higher values of the radiation constants.

The accuracy attained in the final result, estimated from the probable error, is of the order of 0.2 per cent, i. e., the systematic errors are such that the mean values of the various sets of observations differ by 1 to 2 parts in 1000. This means an uncertainty of ± 15 to ± 30 in the value of the constant $C = 14\,450$.

V. EXPERIMENTAL DATA; SERIES OF 1909

This series of observations consists of 34 spectral energy curves obtained with an unpainted porcelain radiator, Fig. 2, which, as in the experiments of Lummer and Pringsheim, was placed directly before the spectrometer slit. The water-cooled shutter and perforated water-cooled diaphragm shown in Fig. 3 were used. The inner heating coil burned out while observing energy curve No. XIX and the radiator was rewound. For the entire research after No. XXV a small knife-edge support was placed under the inner radiator, as explained elsewhere, to prevent it from sagging.

The whole series therefore contains three distinct sets of observations. However, owing to the fact that the radiator was bent from excessive heating and owing to the uncertainty as to the entire elimination of radiation from the side walls and the diaphragms, this series is given no weight in considering the final results.

The spectroradiometric apparatus consisted of an air bolometer (No. 7, resistance of each arm 4.1 ohms) and a small perfectly clear fluorite prism (No. 1, described elsewhere). The bolometer was placed at the focus of a hemispherical mirror 5 cm in diameter. The observations were made under very great difficulties owing to unsteadiness caused by air currents. This made observing almost impossible beyond 4μ in the infra-red, where the deflections were small.

As in each succeeding series, the thermocouples, potentiometer, and standard cell were standardized before and after the series of observations. The mirrors were resilvered at the beginning of each series of observations, so that, in discussing the various series of observations made in subsequent years, no mention will be made of these facts.

In the "series of 1909" observations the radiations from the furnace covered only about one-half of the bolometer strip. In a subsequent work an image of the radiating wall was projected upon and entirely covering the spectrometer slit. In this manner only radiations from the central diaphragm could pass into the spectrometer slit and from thence pass to the bolometer strip which was entirely covered by the incident radiations.

The computations of λ_m by equation (6) give high values, which, for one particular energy curve, the preliminary data of which was published some time ago,[55] gave unusually uniform values of λ_m, for values λ_1 and λ_2 taken over the whole curve. This was one of the few curves completely worked out by the long method of equation (6). All the curves had of course been worked out by equation (1) and, as the title of the paper [55] indicated, it was found that the computations made by equation (1) (the Wien equation) failed to give a uniform value of λ_m computed by taking values of λ_1 and λ_2 over the whole spectrum. In the most recent computations, using a different dispersion (calibration) curve, and including the various correction factors mentioned on a preceding page, it is found that neither the Wien nor the Planck system of computation (equations (1) and (6)) gives uniform values of λ_m. The spectral energy curves appear to bulge upward at 3.5 to 4.5μ. Just how much of this is owing to stray light is difficult to say. As explained elsewhere the whole series is inconsistent with what has been observed since then under more favorable conditions, so that, in the final summary of the work, the numerical results are omitted. However, in view of the discordant results obtained by other experimenters, these data are of interest in showing what great variations may be obtained in the numerical results when using the same prism under various conditions. The results obtained in subsequent years are in excellent agreement. This is especially true if the "series of 1911 and 1912," in which the various sources of error appear to have been eliminated as thoroughly as seems possible with the present type of apparatus. The numerical results of the "series of 1909" are given in Table 9. The data are arranged according to the increase in temperature at which the observations were made. The first column gives the serial number of the experiment. The second column gives the form of the energy curve, "P = Planck," "W = Wien." The "WP" means that the curve lies between the Planck and the Wien equation and is closer to the latter. As explained elsewhere, the criterion for judging the form of the energy curve is the uniformity of the values of λ_m for different values of λ_1 and λ_2 as computed by equation (6) before and after the application of the second term correction factor. If the first term of equation (6) gives uniform values, without deducting the second term, then the curve evidently is of the Wien equation type. If after deducting the second term the values of λ_m are uniform, then the form of the curve is of the Planck equation type; but if, after deducting the second term, the values of λ_m pass through a maximum value (i. e., the second terms overcorrects the results), see Fig. 7, *C*, then it is assumed that the form of the experimental energy curve lies between the Planck and the Wien equations. In the column of λ_m the probable error of the result refers to the departure from the mean value obtained from the number of computations (sixth column) made on any given energy curve.

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[55] Jahrbuch Radioaktivität u. Elektronik, 7, p. 1, 1911, "A characteristic of spectral energy curves."

TABLE 9

Series 1909—Bolometer No. 7, Black Body at Slit, Water-Cooled Shutter No. 1

[Energy Curves, I-XXXV]

No.	Form of energy curve	Absolute temperature	Wave length of maximum emission (λ_{max})			$\lambda_{max}T$	$c=\alpha\lambda_{max}T$
			Mean value	Probable error	Number of computations		
		°	μ	μ			
IX	P	738	3.986	±0.001	4	2942	14 606
X	W	819	3.719	.001	5	3046	15 123
VIII	P+	963	3.233	.015	4	3113	15 458
XI	?	1020	2.984	.017	6	3044	15 112
II	WP	1147	2.671	.012	5	3064	15 211
XV	?	1192	2.560	.005	7	3052	15 151
IV	W?	1229	2.434	.013	4	2991	14 853
XIII	WP	1273	2.351	.005	6	2993	14 860
III	W?	1293	2.295	.008	4	2967	14 733
XII	WP?	1322	2.248	.006	5	2972	14 756
VI	WP	1336	2.238	.006	7	2990	14 846
I	W	1388	2.149	.014	4	2983	14 810
V	W	1420	2.085	.005	5	2961	14 700
VII	P?	1564	1.885	.009	5	2948	14 638
XIV	P+	1624	1.826	.002	7	2965	14 724
XVIII	WP	1667	1.789	.001	6	2982	14 807
XVI	W?	1702	1.756	.004	7	2989	14 839
XVII	P?	1702	1.774	.003	6	3019	14 991
Mean...						3001	14 901
XX	W	777	3.780	±0.004	4	2937	14 583
XXVIII	W?	1073	2.790	.011	3	2994	14 864
XXXII	W−	1192	2.495	.017	5	2974	14 766
XXIX	W?	1270	2.356	.003	6	2992	14 856
XXII	W	1322	2.242	.002	6	2964	14 716
XXXI	PW	1336	2.235	.002	6	2986	14 826
XXI	PW	1358	2.173	.002	5	2951	14 652
XXIII	PW	1420	2.082	.005	7	2956	14 679
XXIV	P?	1516	1.956	.001	5	2965	14 723
XXV	PW	1600	1.850	.004	6	2960	14 697
XXXV	W	1611	1.860	.003	6	2996	14 878
XXVII	WP	1660	1.837	.004	8	3049	15 151
XXVI	W	1667	1.751	.004	5	2919	14 493
XXX	P	1703	1.752	.001	8	2984	14 814
XXXIII	WP	1737	1.734	.001	9	3012	14 955
XXXIV	W−	1765	1.711	.005	7	3020	14 994
Mean...						2979	14 790

In this and in the subsequent tables the values of λ_m are tabulated to the nearest even whole number. Similarly, the values of the temperatures were read to even whole numbers from the thermocouple calibration curves; for it seemed futile to read temperatures to fractions of a degree and include the correction for reduction to the absolute scale when the temperature scale can not be guaranteed to better than 1°.

VI. EXPERIMENTAL DATA; SERIES OF 1910

In the first set of measurements (XXXVI to XL, inclusive) an air bolometer (No. 9, resistance of each strip 6.45 ohms) was used. It was covered with a glass crystallizing dish with a fluorite window for admitting the radiation upon the bolometer strip. The bolometer platinum, being much thinner than in the preceding series, was very seriously affected by air currents, which made it impossible to accurately read the galvanometer deflections.

The second set (XLI to LIV) of energy curves was obtained with a vacuum bolometer (No. 10, resistance of each strip 5.6 ohms) at the focus of a hemispherical mirror. The radiating diaphragm of a new straight porcelain tube was focused upon the spectrometer slit, as shown in Fig. 1. The water-cooled shutter, No. 1, Fig. 3, was placed at *S* in Fig. 1.

Owing to the extreme unsteadiness of the bolometer, the first set of measurements can be given but little, if any, weight in comparison with the rest of the observations. There is also some uncertainty in regard to the temperature scale, which seems to be higher than for the two subsequent years. The sensitivity of the bolometer had a tendency to increase during a series of observations. This, of course, could be adjusted in plotting the curves for computation; but it is never as satisfactory as to have the observations entirely free from changes in sensitivity. Hence, in making up the summary of the most reliable data obtained up to this time, this series is given no weight in comparison with the data obtained in 1912, which, in spite of the small systematic errors for the different sets of observations, are considered the most reliable. The various data are given in Table 10, the explanation of the various columns being the same as for Table 9 of "series of 1909."

TABLE 10

Series 1910, I.—Bolometer in Air, no Hemispherical Mirror

[Energy Curves, XXXVI–XL]

No.	Form of energy curve	Absolute temperature	Wave length of maximum emission (λ_{max})			$\lambda_{max}T$	$C = \alpha\lambda_{max}T$
			Mean value	Probable error	Number of computations		
		°	μ	μ			
XXXIX	?	1180	2.460	±0.008	5	2903	14 413
XXXVIII	W	1278	2.272	.007	4	2904	14 417
XXXVII	W	1341	2.148	.005	6	2880	14 302
XXXVI	P	1423	2.025	.002	5	2882	14 307
XL	P	1528	1.902	.001	7	2906	14 430
Mean...	....	....	....	....	....	2895	14 374

Series 1910, II.—Vacuum Bolometer, Hemispherical Mirror

[Energy Curves, XLI–LIV]

LII[a]	?	1004	2.906	±0.003	4	2918	14 486
XLVII[a]	?	1104	2.642	.010	4	2917	14 482
LI[a]	P	1178	2.472	.005	7	2912	14 458
XLVI	PW	1204	2.441	.001	9	2939	14 592
XLV	P	1280	2.290	.001	5	2931	14 554
L	PW	1305	2.247	.004	8	2932	14 559
XLIV	PW	1341	2.181	.001	8	2925	14 521
XLIII	P	1380	2.145	.001	8	2960	14 697
XLIX[a]	P	1423	2.054	.001	8	2923	14 512
XLII	P	1423	2.058	.002	6	2929	14 540
XLI	P	1480	1.991	.001	7	2947	14 631
XLVIII	P	1504	1.960	.002	9	2948	14 636
LIV	P	1530	1.914	.003	4	2928	14 540
LIII	P	1576	1.873	.002	5	2952	14 656
Mean...	....	....	....	....	....	2933	14 562

[a] Poor curves.

VII. EXPERIMENTAL DATA; SERIES OF 1911

This series (LV to LXXXIV) of spectral energy curves was obtained on an unpainted radiator of Marquardt porcelain, which has a higher melting point and is a better electrical insulator than ordinary porcelain.

The water-cooled shutter, No. 2, Fig. 3, with an opening 4.5 mm in diameter, was attached to the radiator and the whole optical system rendered more air tight, which rendered a greater reduction in humidity.

The potentiometer installation was renovated and tested for insulation. A new fluorite window was placed over the bolometer, and a new resistance was placed in the battery circuit for keeping a constant bolometer current. The bolometer was blackened a little more with soot from a sperm candle. Thin vanes of bolometer platinum were attached to the galvanometer, which completely damped the deflection on a 2.5 seconds single swing. Previously the ballistic throw of the galvanometer was observed. Improvements were also made in the method of viewing the spectrometer circle.

A set of seven isochromatics (LXXXV, 1 to 7) was obtained, which furnished material for computing the constants from spectral-energy curves by means of equation (6). However, owing to the fewness of the observations which makes it difficult to draw an accurate energy curve, this set (LXXXV, 1 to 7) is to be given no weight. The isochromatics appeared unsatisfactory, owing probably to a lack of sufficient time for the radiator to attain a uniform temperature, and the observations were never computed.

The average humidity during the time when this series of observations was obtained was lower than for the series of 1912, and for this reason it was possible to make computations of λ_m with values of λ_z falling at 3.0μ and 3.2μ, which was not possible when the humidity was much higher.

A number of the curves in this series have the property of being slightly higher than required by the Planck equation (for the region beyond 4.8μ), so that the second term factor in equation (6) does not reduce the values of λ_m to a uniform value which is independent of the location of λ_z. The mean value of λ_m is therefore slightly higher than usually obtained. This makes but little difference in the final results; but it is of interest to record the fact, which goes to show how variable are the results obtained with the same apparatus and fluorite prism, in spite of all the precautions taken to eliminate all possible errors. The writer is therefore inclined to believe that some of the discrepancies observed

TABLE 11

Series 1911—Water Cooled Shutter No. 2, Attached to Black Body

[Energy Curves, LV–LXXXIV]

No.	Form of energy curve	Absolute temperature	Wave length of maximum emission (λ_{max})			$\lambda_{max}T$	$C = \alpha\lambda_{max}T$
			Mean value	Probable error	Number of computations		
		°	μ	μ			
LXX [a]	PW	739	3.918	±0.005	3	2895	14 376
LXXI [a]	?	776	3.756	.019	4	2914	14 470
LXXII [a]	?	826	3.530	.009	3	2916	14 478
LXIX	P	865?	3.388	.002	4	2931	14 551
LXVI [a]	PW	1007	2.900	.008	4	2920	14 498
LXVII [a]	PW	1050	2.782	.004	4	2920	14 497
LXVIII [a]	P	1051	2.778	.005	5	2918	14 487
LXI	PW	1150	2.540	.001	5	2921	14 502
LX	P	1201	2.423	.002	6	2909	14 448
LIX	P	1249	2.326	.002	8	2906	14 427
LVI	P	1277	2.274	.002	6	2904	14 418
LVIII	P	1338	2.176	.002	6	2911	14 454
LXV	P	1350	2.150	.001	6	2903	14 412
LXXXIV	PW	1375	2.137	.005	6	2938?	14 589?
LVII	PW	1401	2.080	.002	3	2913	14 466
LV	P	1419	2.057	.002	8	2918	14 490
LXXXIII	P	1474	1.981	.002	7	2920	14 497
LXIV	P	1474	1.978	.001	5	2916	14 478
LXIII	P	1499	1.939	.003	6	2907	14 430
LXII	P	1524	1.910	.003	7	2910	14 445
LXXXII [b]	P	1548	1.898	.004	6	2938?	14 588?
LXXXI	P	1595	1.831	.003	4	2921	14 503
LXXIV	P	1597	1.825	.003	6	2915	14 472
LXXX	P	1620	1.798	.002	4	2912	14 466
LXXIX	P	1648	1.775	.002	4	2923	14 513
LXXVIII [a]	P	1673	1.752	.002	4	2931	14 553
LXXIII	P	1700	1.712	.002	5	2911	14 449
LXXVII [a]	P	1720?	1.721	.007	4	2961?	14 703?
LXXVI [b]	P	1773	1.649	.002	4	2921	14 502
LXXV [b]	PW	1796	1.648	.005	10	2960?	14 696?
Mean...	....	....	....	....	....	2914	14 469
							(14 495)
LXXXV (1) [a]	P	969	3.000	0.003	6	2907	14 434
(2)	?	1139	2.566	.003	3	2923	14 511
(3)	P	1316	2.240	.004	4	2948	14 636
(4)	P	1448	2.020	.002	5	2925	14 523
(5)	P	1564	1.885	.005	4	2948	14 638
(6)	P	1690	1.740	.003	3	2941	14 600
(7)	P	1758	1.673	.001	5	2941	14 603
Mean...	....	....	....	....	....	2933	14 563

[a] Poor curves. [b] Bolometer sensitivity varied.

by other experimenters when modifying their apparatus may be due to these systematic errors and not due entirely to the different prisms used.

The data for the "series of 1911" are given in Table 11, the arrangement of this table being the same as for the "series of 1909" given on a preceding page. The results of this series are in excellent agreement with the subsequent data (series of 1912), which was obtained with entirely new adjustments. Considerable confidence is therefore placed in these two series of observations. The mean value of all the energy curves gives $C = 14\ 495$. If we exclude the four energy curves indicated (?) in Table 11, for which the values of C are extraordinarily high, the mean value is but slightly reduced, being $C = 14\ 469$. The writer feels justified in omitting these four high values because in at least two cases (LXXV and LXXXII) the bolometer was freshly exhausted and the residual gases were changing from hydrogen (and nitrogen) to the characteristic carbon dioxide spectrum. The radiation sensitivity, therefore, probably changed more than indicated by direct experiment. For, as already explained, the radiation sensitivity of the bolometer tended to decrease and the galvanometer sensitivity tended to increase, so that after such a prolonged series of observations these variations may have escaped notice.

VIII. EXPERIMENTAL DATA; SERIES OF 1912

1. **White Radiating Walls; Prism No. 1.**—After readjusting the apparatus throughout, a series of observations (LXXXVI to XCII) was made upon the unblackened Marquardt tube used the preceding year, to obtain a check upon the previous work and also upon the work that was to follow. The fluorite prism, No. 1, was reset, and minor changes were made in the entire apparatus. Unless otherwise stated, the water-cooled shutter No. 2 was used throughout this series. The data are given in Table 12, the mean value being $C = 14\ 463$.

TABLE 12

Series 1912, I—White Radiating Walls, Fluorite Prism No. 1, Water-Cooled Shutter No. 2

[Energy Curves, LXXXVI–XCII]

No.	Form of energy curve	Absolute temperature	Wave length of maximum emission (λ_{max})			$\lambda_{max}T$	$C=\alpha\lambda_{max}T$
			Mean value	Probable error	Number of computations		
		°	μ	μ			
XCII [a]	?	1122	2.605	±0.005	3	2922	14 508
LXXXVIII	P	1209	2.408	.003	6	2910	14 451
XCI [a]	P	1223	2.374	.002	4	2904	14 418
LXXXVII	PW	1242	2.345	.001	5	2912	14 459
LXXXVI	P	1350	2.154	.002	6	2908	14 438
LXXXIX [b]	P	1419	2.056	.003	7	2916	14 480
XC [a]	PW	1427	2.045	.006	5	2918	14 486
Mean						2913	14 463

[a] Furnace heated by outer coil only.

[b] Poor curve.

2. **Black Radiating Walls.**—For this set of observations (XCIII to CVI) the inner porcelain tube was painted with a thick coat of chromium oxide. Owing to the smallness of the opening, the inner tube, *A*, Fig. 4, had to be removed and the painting done in strong sunlight. The pigment was mixed with water and applied with a fine-haired brush. The front thermocouple rested in contact with the chromium oxide. At the end of the series no effect of the oxide upon the thermocouple could be observed. The humidity was lower than usual for this series. The water-cooled shutter No. 2 was used. The temperature of the water, just after leaving the perforated diaphragm, was 9.4° when the radiator was at a temperature of 1100°. Since the thin metal door, which permitted the passage of radiations from the furnace to the spectrometer, was directly in contact with the water-cooled diaphragm, it was assumed that (as on previous tests) the temperature of the door was not much above that of the flowing water. This question is discussed elsewhere. The data of this series are given in Table 13, the mean value being $C = 14\,432$.

TABLE 13

Series 1912, II—Black (Cr_2O_3) Radiating Walls, Fluorite Prism No. 1, Water-Cooled Shutter No. 2

[Energy Curves, XCIII–CVI]

No.	Form of energy curve	Absolute temperature	Wave length of maximum emission (λ_{max})			$\lambda_{max}T$	$C=\alpha\lambda_{max}T$
			Mean value	Probable error	Number of computations		
		°	μ	μ			
CVI [a]	P	1025	2.824	±0.001	5	2895	14 372
XCIX	P	1153	2.507	.002	4	2891	14 356
XCVIII [a]	P	1223	2.375	.004	4	2904	14 419
XCVII	P	1301	2.226	.002	6	2896	14 379
CV	P	1338	2.183	.002	5	2921	14 503
XCV [a]	P	1351	2.145	.001	4	2898	14 388
XCVI	P	1377	2.104	.003	4	2897	14 385
CIV	P	1401	2.080	.003	6	2914	14 470
XCIV	P	1420	2.050	.004	7	2909	14 442
CII	P	1453	2.003	.008	7	2910	14 450
CIII	P	1479	1.973	.004	5	2918	14 491
XCIII	P	1501	1.944	.004	4	2917	14 485
CI	PW	1525	1.908	.005	5	2910	14 447
C	P	1687	1.726	.002	5	2912	14 457
Mean...	....	....	....	....	....	2907	14 432

[a] Poor curves.

3. **New Radiator; White Walls.**—This set of energy curves (CVII to CXIII) was obtained with a new unpainted Marquardt porcelain tube. The platinum ribbon was 20 mm wide at the center and decreased uniformly to 11 mm in width at the ends, which is somewhat narrower than for the preceding radiator. The end diaphragm was broken away, as shown in Fig. 4, *B*, so as to permit of painting the inner walls without removing the tube. The end was, of course, closed with a similar porcelain diaphragm when ready for observations. The thermocouple was entirely inclosed in a small unpainted porcelain tube, as shown in Fig. 4, *B*. The axial adjustment of the radiator was made anew; and the optical path was traced and adjusted throughout the whole apparatus by the light coming from the radiator heated to 1300°. The water-cooled shutter No. 2 and fluorite prism No. 1 were used.

The data of this series are given in Table 14, the mean value being $C = 14\ 466$.

TABLE 14

Series 1912, III—New Radiator, White Walls, Fluorite Prism No. 1, Water-Cooled Shutter No. 2

[Energy Curves, CVII–CXIII]

No.	Form of energy curve	Absolute temperature	Wave length of maximum emission (λ_{max})			$\lambda_{max}T$	$C = \alpha\lambda_{max}T$
			Mean value	Probable error	Number of computations		
		°	μ	μ			
CXIII [a]	?	1151	2.540	±0.002	3	2921	14 504
CX	PW	1223	2.383	.003	6	2914	14 469
CXII	P	1276	2.275	.001	6	2903	14 412
CIX	P?	1352	2.155	.002	7	2914	14 466
CXI	P	1379	2.109	.004	5	2911	14 451
CVIII	P	1453	2.003	.003	5	2910	14 450
CVII	P	1527	1.913	.003	4	2921	14 508
Mean...	....	....	....	....	....	2913	14 466

[a] Poor curve.

4. **New Radiator; Black Walls.**—For this set of energy curves (CXIV to CXXIII) all the adjustments and accessory apparatus were the same as for the preceding series. The inner walls, the front side of the radiating diaphragm, and the short tube covering the thermocouple were painted with a layer of chromium oxide. On this layer was painted a thick coat of cobalt oxide. It was found that the inclosed thermocouple responds almost as quickly to temperature changes as obtained in the bare thermocouple. At the completion of the set it was found that some of the oxide had become detached from the short porcelain tube covering the thermocouple, which left this tube a bluish white. In spite of this fact, however, the percentage of curves fitting the Planck equation is noticeably larger for the blackened than for the unblackened tubes.

The data of this series are given in Table 15, the mean value being $C = 14\ 459$, or if we exclude the last value, which is extraordinarily high (a poor curve), the mean value is practically unchanged, being $C = 14\ 446$.

TABLE 15

Series 1912, IV—New Radiator, Black ($Cr_2O_3+2Co_2O_3$) Walls, Fluorite Prism No. 1, Water-Cooled Shutter No. 2

[Energy Curves, CXIV–CXXIII]

No.	Form of energy curve	Absolute temperature	Wave length of maximum emission (λ_{max})			$\lambda_{max}T$	$C=\alpha\lambda_{max}T$
			Mean value	Probable error	Number of computations		
		°	μ	μ			
CXXIII[a]	P?	876	3.312	±0.004	3	2901	14 405
CXXII	P	1025	2.826	.002	8	2897	14 382
CXXI	P	1077	2.704	.005	5	2912	14 459
CXVIII	P	1154	2.516	.003	6	2903	14 414
CXX	PW	1223	2.376	.001	5	2906	14 428
CXVII	P	1276	2.280	.001	5	2909	14 445
CXIX	P	1353	2.156	.002	6	2917	14 486
CXVI[a]	P	1377	2.122	.007	6	2922	14 507
CXV	P	1454	2.007	.005	6	2918	14 488
CXIV[a]	P	1503?	1.954	.004	4	2936	14 580?
Mean	....	....	....	....	....	2910	14 446 (14 459)

[a] Poor curves.

5. **Blackened Radiator; New Adjustments.**—In this series (CXXIV to CXL) the short porcelain tube inclosing the thermocouple was covered with iron oxide (from writing ink, which was then burned into the tube by means of a blast lamp) over which was painted a layer of chromium oxide. To do this, the water-cooled shutter had to be removed. This series therefore differs from the preceding only in that the radiator was repainted with a mixture of chromium and cobalt oxides and the complete optical system was readjusted. At the highest temperatures (1300° to 1500°, series CXXXVIII to CXL) the radiator was heated by electric current from a motor-generator operated on a storage battery. For temperatures up to 1250° there was little or no conduction of current from the heating coils to the thermocouples.

The data of this series are given in Table 16, the mean value being $C = 14\ 501$. If we exclude the three (indicated in Table 16) which have either a high probable error for the λ_m, which are poor curves or were obtained at such high temperatures that the use of a fluorite prism is questioned, the value is $C = 14\,476$.

TABLE 16

Series 1912, V—Blackened Radiator, New Adjustments, Fluorite Prism No. 1, Water-Cooled Shutter No. 2

[Energy Curves, CXXIV–CXL]

No.	Form of energy curve	Absolute temperature	Wave length of maximum emission (λ_{max})			$\lambda_{max}T$	$C = \alpha\lambda_{max}T$
			Mean value	Probable error	Number of computations		
		°	μ	μ			
CXXXV[a]	?	1026	2.831	±0.005	5	2905	14 423
CXXXII[a]	P	1077	2.703	.004	5	2911	14 455
CXXXIV	PW	1153	2.520	.003	6	2905	14 425
CXXXI	P	1194	2.441	.003	4	2914	14 469
CXXIX	P	1224	2.380	.001	6	2913	14 464
CXXVI	P+	1276	2.284	.006	9	2914	14 469
CXXXIII	P+	1302	2.237	.002	6	2913	14 461
CXXXVII	PW	1302	2.242	.003	7	2920	14 498
CXXX	P	1338	2.182	.003	5	2920	14 497
CXXVIII	P+	1352	2.160	.004	4	2921	14 501
CXXV	P	1378	2.118	.004	5	2918	14 483
CXXXVI	P	1421?	2.068	.003	5	2939	14 588?
CXXVII	P+	1454	2.008	.004	9	2920	14 497
CXXIV	P+	1503	1.961?	.007	4	2948	14 629?
CXL[b]	P	1627	1.798	.003	5	2925	14 524
CXXXVIII[b]	PW	1663	1.756	.004	5	2921	14 500
CXXXIX[ab]	P+	1796	1.639	.002	8	2945	14 631?
Mean...	....	....	....	....	....	2915	14 476 (14 501)

[a] Poor curves.

[b] Alternating current used for furnace heating coils.

6. **Blackened Radiator; Prism No. 2.**—In this set of observations (CXLI to CXLVII) all the adjustments were the same as in the preceding series. The small fluorite prism was replaced by a large one, No. 2, described elsewhere. This was for the purpose of testing the effect of the prism, all other adjustments remaining the same. The humidity was lower than for the preceding series, so that the distortion of the energy curves in the region of 3.5 to 4μ is not due to atmospheric absorption beyond 5μ, but is no doubt

owing to defects in the structure of the prism, which visually caused much more stray light than in a clear prism. The spectral energy curves obtained with this prism are entirely different from those obtained with a clear prism. As shown in Table 17, only about 30 per cent of the energy curves fall within the limits set for agreement with the Planck equation.

TABLE 17

Series 1912, VI—Blackened Radiator, Fluorite Prism No. 2, Water-Cooled Shutter No. 2

[Energy Curves, CXLI–CXLVII]

No.	Form of energy curve	Absolute temperature	Wave length of maximum emission (λ_{max})			$\lambda_{max}T$	$C = \alpha\lambda_{max}T$
			Mean value	Probable error	Number of computations		
		°	μ	μ			
CXLII [a]	W	1077	2.745	±0.007	4	2956	14 679
CXLVII	PW	1154	2.580	.001	9	2977	14 783
CXLVI	W	1194	2.475	.002	9	2955	14 673
CXLI	P	1225	2.402	.004	7	2942	14 610
CXLV	P	1277	2.331	.003	9	2977	14 780
CXLIV	P?	1352	2.188	.001	10	2958	14 688
CXLIII	P	1454	2.055	.001	8	2988	14 836
Mean...	....	....	....	....	....	2969	14 721

[a] Poor curve.

7. **New Thermocouple; Prism No. 2.**—The object of this series (CXLVIII to CLV) was to test the effect of a change in thermocouples, the large fluorite prism, No. 2, and the other accessories remaining unchanged. The thermocouple was entirely new, and unused except for obtaining its calibration. The thermoelement at the rear of the radiating diaphragm was recalibrated. The coating of cobalt oxide, which was on the short porcelain tube inclosing the thermoelement, was rendered more adhesive by mixing the pigment with a small quantity of kaolin. This was burned upon the tube before mounting the thermocouple in the radiator, which was then repainted before resetting the water-cooled shutter.

As in the preceding series, only a small proportion of the energy curves fit the Planck equation. The data of this series are given in Table 18.

TABLE 18

Series 1912, VII—Blackened Radiator, New Thermocouple, Fluorite Prism No. 2, Water-Cooled Shutter No. 2

[Energy Curves, CXLVIII–CLV]

No.	Form of energy curve	Absolute temperature	Wave length of maximum emission (λmax)			λ_{max}T	C=$\alpha\lambda_{max}$T
			Mean value	Probable error	Number of computations		
		°	μ	μ			
CLII[a]	W	1078	2.727	±0.003	3	2940	14 596
CLV	WP	1195	2.467	.006	5	2948	14 638
CLI	WP	1223	2.403	.003	10	2939	14 592
CL	PW	1277	2.301	.002	10	2938	14 589
CLIV	P	1301	2.271	.001	8	2955	14 670
CXLIX	PW	1353	2.180	.001	10	2950	14 645
CLIII	WP	1377	2.145	.003	9	2954	14 665
CXLVIII	P	1453	2.035	.002	7	2957	14 681
Mean...	....	....	....	....	....	2947	14 634

[a] Poor curve.

8. **New Thermocouple; Prism No. 1.**—This set (CLVI to CLXVII) of energy curves was obtained with the small fluorite prism, No. 1, to test the effect of a small displacement, 58′, of its center of rotation upon the prism table. This is not a very important factor, but it seemed worth while to test it. The rest of the apparatus, including the thermocouple, was the same as in the preceding experiment. The optical path was, of course, tested as usual for adjustment upon the spectrometer slit, upon the prism face, and upon the bolometer. The result of this test show that within the usual experimental errors of observation and variation in humidity there is no deviation in the values of the constants which can be attributed to a displacement of the axis of rotation of the prism mirror system.

This and the following series of observations were made under more unfavorable conditions than usually encountered. Curve CLXII was poor owing to unsteadiness of the bolometer, caused by a low vacuum; and the last day's work (CLXV to CLXVII) was spoiled by tremors which at times made it impossible to read the galvanometer. While observing a spectral energy curve the platinum ribbon on the inner radiator became disrupted, so that curves CLXVI and CLXVII were obtained by heating the inner tube, Fig. 2, by radiation from the outer heater. No difference is to be observed in its effect upon numerical results as compared with curve CLXV, which was obtained on the same day. There was, of course, a greater lag in controlling the temperature when heating the inner tube by radiation than when heating it by means of the platinum ribbon with which it was surrounded.

The data of this series are given in Table 19, the mean value being $C = 14\ 518$, or if we exclude the last three in the series, which evidently can not be given equal weight with the rest, the mean value is $C = 14\ 461$.

TABLE 19

Series 1912, VIII—Blackened Radiator, New Thermocouple, Fluorite Prism No. 1, Water-cooled Shutter No. 2

[Energy Curves, CLVI–CLXVII]

No.	Form of energy curve	Absolute temperature	Wave length of maximum emission (λ_{max})			$\lambda_{max}T$	$C = \alpha\lambda_{max}T$
			Mean value	Probable error	Number of computations		
		°	μ	μ			
CLIX[a]	?	806	3.610	....	....	2910	14 447
CLX	WP	879	3.300	±0.003	2	2901	14 402
CLXI[b]	P	953	3.052	.003	2	2909	14 441
CLVIII	P	1077	2.703	.002	7	2911	14 454
CLVII[c]	P?	1223	2.380	.004	6	2911	14 453
CLVI	P	1277	2.286	.003	7	2920	14 497
CLXIV[d]	P	1300	2.248	.004	8	2923	14 513
CLXVII[eb]	WP	1314	2.236	.003	7	2938?	14 588?
CLXIII	P	1376	2.120	.004	6	2917	14 482
CLXVI[bd]	P	1377	2.140	.002	5	2947?	14 634?
CLXII[f]	P+	1452	2.030	.004	10	2948?	14 639?
CLXV[db]	P+	1539	1.920	.003	9	2955?	14 671?
Mean...	....	....	....	....	....	2912	14 461 (14 518)

[a] Very poor curve, λ max by inspection.
[b] Poor series, tremors.
[c] Poor curve.
[d] Radiation sensitivity changed.
[e] Furnace heated by outer coil only.
[f] Bolometer unsteady; low vacuum.

9. **Water-Cooled Shutter No. 1; Prism No. 1.**—In this set (CLXVIII to CLXXV) the water-cooled shutter, No. 1, and the separate water-cooled diaphragm, Fig. 3, with an opening 6.7 mm. (placed at *S*, Fig. 1) was used with the object of testing the effect of the size of the opening in the water-cooled diaphragm, upon the energy curves. It was not possible to prevent, so effectively as with shutter No. 2, the entrance of air into the spectrometer boxes. The humidity was higher than in any of the preceding sets of observations, the sensitivity of the galvanometer was continually changing while observing several of the energy curves, and occasional tremors caused by an unbalanced reciprocating engine sometimes prevented an accurate reading of the galvanometer, so that this series should perhaps not be given as great as some of the preceding sets of energy curves obtained with this same fluorite prism No. 1. This series, however, was observed under more favorable conditions than obtained during the latter part of the preceding series. The radiator was the same as in the

TABLE 20

Series 1912, IX—Blackened Radiator, Fluorite Prism No. 1, Water-cooled Shutter No. 1

[Energy Curves, CLXVIII–CLXXVI]

No.	Form of energy curve	Absolute temperature	Wave length of maximum emission (λ_{max})			$\lambda_{max}T$	$C=\alpha\lambda_{max}T$
			Mean value	Probable error	Number of computations		
		°	μ	μ			
CLXXIV [a]	P?	1175	2.465	±0.004	7	2896	14 381
CLXXIII	P	1194	2.431	.002	4	2900	14 401
CLXXV	P	1277	2.272	.003	5	2901	14 405
CLXXII	P	1302	2.232	.002	4	2906	14 429
CLXXVI	P	1353	2.153	.001	7	2913	14 462
CLXXI	P	1377	2.114	.001	7	2911	14 456
CLXX	P	1451	2.010	.002	7	2916	14 477
CLXIX [a]	P	1452	2.007	.003	2	2914	14 467
CLXVIII [a]	P	1540	1.890	.003	2	2909	14 443
Mean...	....	....	....	....	....	2909	14 436

[a] Poor curves, sensitivity changing.

preceding sets (Nos. 7 and 8), the break in the platinum ribbon having been repaired, and the optical system having been readjusted before the observations were resumed. A geometrical diagram of the optical path of the radiations shows that, owing to the large opening in the water-cooled diaphragm, it is possible for radiations from the walls of the furnace to enter the spectrometer slit. However, the higher humidity and other difficulties encountered in this series of observations were probably sufficient to mask any errors resulting from extraneous radiations. This series of energy curves gives very uniform results, just as though the increased humidity within the spectrometer box had not been deleterious.

The data of this series are given in Table 20, the mean value being $C = 14\,436$.

IX. SUMMARY

The present paper gives data relating principally to the determination of the mathematical form of the isothermal spectral curve of a black body, and to the computation of the numerical values of the constants relating thereto, for the temperature range between 450° and 1525° C.

This paper contains also data on various subsidiary problems such as (1) the variation of the reflecting power of silver with angle of incidence and with wave length, (2) the variation of reflecting power of fluorite with angle of incidence and with wave length (refractive index), (3) data on the most reliable refractive indices of fluorite, reduced to 20° C, and (4) data for reducing the observations from prismatic to normal spectrum.

The spectrum was produced by means of a mirror spectrometer and a fluorite prism. A vacuum bolometer was used for measuring the partition of energy in the spectrum. The radiator was a porcelain tube, wound with platinum ribbon through which electric current was passed. The various incidental questions such as the adjustments of the optical parts of the apparatus, the temperature uniformity and control of radiator, the water-cooled shutters, the temperature scale, the method of reduction of the observations to the normal spectrum, the proper formulæ for computing the numerical constants, etc., are also discussed.

In a brief summary the results of previous investigations made with similar apparatus are discussed. It is shown that by the elimination of certain errors in the calibration of the prisms used, the older observations are not in disagreement, as was supposed from the published results. It is also shown that, if these older data had been computed by the methods employed in the present paper, the numerical values of the constants would, without doubt, be in very close agreement with the present results. These older data (properly weighted) are therefore used in connection with the present-day results, to form an estimate of the most probable value of the constants of spectral radiation.

The results obtained in the present investigation are based upon experiments with different bolometers, water-cooled shutters, fluorite prisms, thermocouples, and (blackened and unblackened) radiators. It is felt that what success has been attained is owing to blackening the radiator, to properly insulating the thermocouples within the radiator, to placing a knife-edge support beneath the inner radiator to prevent it from sagging, and to projecting an image of the radiating diaphragm upon the spectrometer slit. It is shown that the percentage of the total number of energy curves which fit the Planck equation is increased by blackening the radiator.

About 75 per cent of the reliable energy curves fit the Planck equation, the validity of which was assumed in order to compute the radiation constants from the observed data. This involves a temperature range greater than 1000°. Within this temperature range there is a tendency for the constants to increase slightly with temperature, in several sets of observations, but owing to the various difficulties involved in the work, this slight variation is not sufficiently systematic to be considered an actual departure from the theoretical (Planck) law, upon which the computations are based.

In Table 21 are assembled the reliable data which form the basis for obtaining a mean value of the radiation constants. The mean value of these eight sets (which among themselves contain the mean values of 94) of the best spectral energy curves is:

$$C = 14\,456 \pm 4 \text{ mikron deg.}$$
$$A = 2911 \pm 1 \text{ mikron deg.}$$

The mean value of these 94 energy curves, taken as a single group, hence giving the same weight to each curve, is $C = 14\,458$. The mean value of all energy curves observed in 1911 and 1912 (the 105 energy curves in the eight sets of observations, not excluding the poor ones) is $C = 14\,471 \pm 8$. Here the probable error of the result is doubled by including the doubtful curves which should be rejected for the reasons given in the text. However, it is evident that no matter how we combine the observations, it affects but little the mean value of the constant. This is owing to the large number of energy curves, and the large number of computations (4 to 7) made upon each energy curve. The final value is, in fact, based upon about 600 individual values.

TABLE 21

Series	Number of energy curves	Mean value C	Remarks
1911			
I	26	14 469	Unpainted Marquardt porcelain radiator. (Water-cooled shutter No. 2.)
1912			
I	7	14 463	Unpainted radiator as used in 1911.
II	14	14 432	Radiator painted with chromium oxide.
III	7	14 466	New unpainted Marquardt radiator.
IV	9	14 446	Radiator painted with a mixture of the oxides of cobalt and chromium.
V	14	14 476	Radiator repainted; optical path readjusted.
VIII	8	14 461	New thermocouples; radiator the same as in the preceding series.
IX	9	14 436	New water-cooled shutter, No. 1; the rest of the apparatus is the same as in the preceding series.

Mean value $C = 14\,456 \pm 4$ (94 energy curves)
$A = 2911 \pm 1$

The weighted mean value of the four sets of observations mentioned in the text (viz, Lummer and Pringsheim,[56] $C = 14\,460 \times 1$; Paschen,[56] $C = 14\,458 \times 3$; Coblentz, $C = 14\,456 \times 10$; Warburg, $C = 14\,374 \times 10$) is $C = 14\,420$ and $A = 2905$. The mean value of the results of Warburg and of the writer is $C = 14\,415$.

The difference in the results of the two recent determinations causes an uncertainty of about 3° at the melting point of palladium (m. p. = 1549°) and of about 5° at the melting point of platinum (m. p. = 1755°).

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[56] After recomputation as explained.

In conclusion, especial acknowledgment is due Mr. U. F. Rosen for the numerical computations involved in this research.

WASHINGTON, December 12, 1912.[57]

X. NOTE ON THE TEMPERATURE SCALE.

During the past month Dr. Burgess and Mr. Kellberg have made a thorough investigation of the thermocouples used in this research, including a calibration at the melting point (1549°) of palladium. This was done in order to obtain a check upon the accuracy of the temperature scale obtained by extrapolation from the melting point (1083°) of copper, as described on a previous page.

The annealed thermocouples (known as C_2, C_3, C_4) were first calibrated by freezing and melting points in copper, antimony, and zinc. The emfs observed were somewhat lower (equivalent to 0°.5 to 1° in temperature) than the previous calibrations. However, this does not require a correction in the temperatures recorded in this investigation, for in the meantime the thermocouples had been used in determining spectral energy curves produced by a cloudy fluorite prism (to be mentioned presently), after which the radiator was heated above 1500° for some tests with a colorimeter. The high temperatures employed in the latter work were no doubt responsible for the change in the emf of the couples.

A homogeneity test was then made on these couples, one of which showed a contaminated place in the platinum wire. However, since the mean value of the results obtained with this thermocouple are in agreement with the values obtained with the other thermocouples, no apprehension is felt because of this fact.

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[57] This is the date when the writing of this paper was completed and abstracts were issued for publication (J. Wash. Acad. Sci., 3, p. 10, 1913; and Amer. Phys. Soc. (Phys. Rev. (2), 1, p. 249, 1913)). As explained in the text (see "Temperature scale") there was a systematic increase in the value of C for temperatures above 1400°; and it was necessary to assume this to be a fault in the estimation of temperatures or in the Planck equation. The writer chose the former alternative and modified the corrections to the optical temperature scale. This modification of the temperature scale above 1400° has no effect upon the data obtained in 1912, which is the crucial work, owing to the fact that intentionally (to save the thermocouple calibration, the difficulty with the temperature correction was not foreseen) no energy curves were observed at high temperatures. The systematic errors which formerly caused the mean values of the various sets of observations to differ by 0.2 to 0.5 per cent are now reduced to less than 0.2 per cent. The former values of C published in these two journals are therefore withdrawn. Washington, January 15, 1913.

The couples were then annealed by heating them electrically to about 1600° C. The palladium points were then taken on C_2 and C_3 by fusing a small piece of palladium about 5 mm long between the platinum and the alloy wires and melting the palladium in an iridium tube furnace. The couples were then calibrated at the freezing points of copper, antimony, and zinc. The couple C_3 showed no change as the result of annealing. This couple is in close agreement with the Geophysical Laboratory's standard curve given in the American Journal of Science, July, 1910.

The palladium used in this calibration was obtained from Heraeus, and from Day & Sosman of the Geophysical Laboratory. The results from the two kinds of material are in agreement within the limits of experimental error.

As explained in Part IV of this paper, the true temperatures were obtained by adding to the temperatures on the thermoelectric scale certain values given in Table 4. The present data, including the calibration at the melting point of palladium, were obtained to test the accuracy of the temperature corrections given in Table 4. For this purpose the thermocouple calibration curve was computed using zinc, antimony, and copper as the three fixed points. The calibration curve was computed also by using antimony, copper, and palladium as the fixed points. The difference between the calibration curve extrapolated beyond the copper fixed point and the calibration curve obtained by interpolation between the copper and the palladium fixed points gives the true temperature corrections. The results obtained with the two thermocouples show that the corrections in Table 4 are too small at the lowest temperatures (1,200°) and too large at the highest temperatures. The correction is about 2° at 1,200° and about 8° at 1,400°. Making these corrections in Tables 11 to 20, inclusive, has but little effect on the final mean value. For example in Table 21 the mean values of C taken in succession are to be increased by the following values: 22, 4, 14, 7, 2, 9, 0, 14. This increases by 9 the mean value (C = 14,456) of the 8 sets of observations, so that the corrected mean value is

$$C = 14{,}465.$$

The results show that these corrections have but little effect upon the final mean value.

The latest results of Hoffmann & Meisner[58] on the ratio of brightness of two black bodies at the melting points of gold and of palladium, using an optical pyrometer, gives a value of C = 14,440. The value now commonly used is C = 14,500, and from present indications there appears to be no need of haste in adopting a new value.

In conclusion it is desirable to record the results of an investigation made during the past year of the shape of the spectral energy curves obtained with a new fluorite prism having faces 30 by 34 mm. and an angle of 60°. The prism was white and had several streaks of cloudiness but no internal cleavage faces. The energy curves seemed distorted similar to the ones recorded for prism No. 2, mentioned in this paper, and the constants were as high in value as those found for the latter prism (see Table 17). It therefore appears that the homogeneity of the prism is an important factor in determining the shape of an energy curve.

WASHINGTON, October 9, 1913.

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[58] Zeit. für Instrumentenkünde 33, p. 157, 1913.